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Note on J-groups of Lens Spaces

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§1. Introduction

Let $J(X)$ be the J-group of a CW-complex X of finite dimension. Then by J. F. Adams [2] and D. Quillen [9], it is shown that

(1.1) $$J(X) = KO(X)/\mathrm{Ker}\,J, \qquad \mathrm{Ker}\,J = \textstyle\sum_k(\cap_e k^e(\Psi^k - 1)KO(X)),$$

where $KO(X)$ is the KO-group of real vector bundles over X, $J: KO(X) \to J(X)$ is the natural epimorphism and Ψ^k is the Adams operation.

In this note, we consider the standard lens space

$$L^n(m) = S^{2n+1}/Z_m, \qquad Z_m = \{\exp(2\pi li/m): 0 \leqq l < m\},$$

which has the CW-decomposition $L^n(m) = \cup_{j=0}^{2n+1} e^j$ and its $2n$-skeleton

$$L_0^n(m) = \{[z_0,\dots, z_n] \in L^n(m): z_n \text{ is real} \geqq 0\}.$$

Let η be the canonical complex line bundle over $L^n(m)$ or $L_0^n(m)$. Then

(1.2) (N. Mahammed [7]) *the K-rings of complex vector bundles over these spaces are given by*

$$K(L^n(m)) = K(L_0^n(m)) = Z[\eta]/ < \eta^m - 1, (\eta - 1)^{n+1} >,$$

and the reduced group $\tilde{K}(L^n(m)) = \tilde{K}(L_0^n(m))$ *is of order* m^n,
where $Z[\eta]$ is the integral polynomial ring with one variable η and the denominator is the ideal generated by $\eta^m - 1$ and $(\eta - 1)^{n+1}$.

Now, we consider the case $m = p^r$, where p is an *odd prime* and $r \geqq 1$. Then by (1.2) and (1.1), we have the following

PROPOSITION 1.3. (i) $$J(L^n(p^r)) = \begin{cases} J(L_0^n(p^r)) \oplus Z_2 & \textit{if } \; n \equiv 0 \bmod 4, \\ J(L_0^n(p^r)) & \textit{otherwise.} \end{cases}$$

(ii) $$Jr: K(L_0^n(p^r)) \xrightarrow{r} KO(L_0^n(p^r)) \xrightarrow{J} J(L_0^n(p^r))$$

(r *is the real restriction*) *is epimorphic, and* Ker Jr *is the subgroup of* $K(L_0^n(p^r))$ *of* (1.2) *for* $m = p^r$ *generated additively by the elements*