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A Boundary Value Problem for Delay Differential Equtions

Shigemi OHKOHCHI

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1. Introduction

In this brief paper, we shall give a result on the existence of solutions of a boundary value problem for second order delay differential equations. We consider the delay differential equation

$$(1)\qquad (\rho(t)x'(t))' = f(t, x(t), x(t-g(t)), x'(t)), \qquad a \leqq t \leqq b$$

with boundary conditions

$$(2)\qquad x(t) = \phi(t), \qquad t \leqq a, \quad x(b) = A,$$

where $f(t, x, y, z)$ and $\phi(t)$ are continuous functions defined on $[a, b] \times R^3$ and $(-\infty, a]$, R being the real line, respectively, $\rho(t)$ is a positive, continuously differentiable function defined on $[a, b]$ and A is an arbitrary constant. In (1), the lag $g(t)$ is assumed to be a nonnegative continuous function defined on $[a, b]$.

Several authors have contributed to the establishment of the existence and uniqueness of solutions of such boundary value problems. Among them, K. de Nevers and K. Schmitt [1] have obtained an existence theorem of a unique solution under the assumption that $\rho(t) \equiv 1$ and the right hand member f of (1) does not depend on the fourth argument $x'(t)$ and satisfies the following condition:

$$(3)\qquad \begin{aligned} &f(t, x, y) - f(t, \bar{x}, \bar{y}) \geqq p(t)(x-\bar{x}) - q(t)(y-\bar{y}) \\ &\quad \text{if} \quad x \geqq \bar{x},\ y \geqq \bar{y} \quad \text{and} \quad t \in [a, b], \end{aligned}$$

where $p(t)$ is a continuous function and $q(t)$ is a nonnegative continuous function. Their proof is based on the so-called shooting method.

We here apply the same shooting method to prove our existence theorem, though our hypotheses on f are somewhat different from those of K. de Nevers and K. Schmitt and, for instance, the condition (3) may be replaced by the following:

$$f(t, x, y, z) - f(t, \bar{x}, \bar{y}, \bar{z})$$