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Some Remarks on the Global Transforms of Noetherian Rings

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In his paper [4], Matijevic generalized the Krull-Akizuki theorem on the intermediate rings between a noetherian domain of Krull dimension one and its quotient field to the case of general noetherian rings, using the notion of the global transform of a noetherian ring. The aim of this paper is to give some interesting properties of the global transforms of noetherian rings. For a noetherian ring A, the global transform A^g is the set $\{x \in Q(A);\ x \in A \text{ or } \dim(A/(A:x))=0\}$, where $Q(A)$ is the total quotient ring of A. Now A^g coincides with the $\mathscr{C}$-divisorial envelope of A in $Q(A)$, where $\mathscr{C}$ is the Serre subcategory of $\mathrm{Mod}(A)$ consisting of all A-modules M such that $\mathrm{Supp}(M) \subseteq \mathrm{Max}(A)$ (for $\mathscr{C}$-divisorial envelope, see [2]). On the other hand, a $\mathscr{C}$-divisorial module M is characterized by $\mathrm{Ext}_A^1(N, M)=0$ for every object N in $\mathscr{C}$. In other words, the relation between A^g and A depends deeply on the set $\{\mathrm{depth}(A_{\mathfrak{m}});\ \mathfrak{m} \in \mathrm{Max}(A)\}$. Among our results in this paper, we shall show that the canonical homomorphism $A \to A^g$ is a flat epimorphism for any noetherian normal domain A, and also that the global transform of A^g is A^g itself if A is reduced. Finally we give an example which shows that the Corollary to the Theorem in [4] is not true if we drop the assumption of reducedness of A.

Throughout this paper, all rings are commutative with unit. We use the following notations: for a ring A,

$\mathrm{Max}(A)$ = the set of all maximal ideals in A,

$z(A)$ = the set of all zero divisor of A,

$(A:x)=\{a \in A;\ ax \in A\}$, where x is any element of $Q(A)$.

PROPOSITION 1. *Let A be a noetherian ring. Then the following statements hold;*

a) *If $\dim(A) \leq 1$, then $A^g = Q(A)$.*

b) *$S^{-1}(A^g) \subseteq (S^{-1}A)^g$ holds for any multiplicatively closed subset S of A.*

c) *Suppose that $ht(\mathfrak{p}) \leq 1$ for any associated prime ideal $\mathfrak{p}$ in A. Let S be a multiplicatively closed subset of A such that $\mathrm{Max}(S^{-1}A) \subseteq ({}^a i)^{-1}(\mathrm{Max}(A))$, where ${}^a i$ is the morphism of $\mathrm{Spec}(S^{-1}A)$ to $\mathrm{Spec}(A)$ defined by the canonical homomorphism i of A to $S^{-1}A$. Then $S^{-1}(A^g) = (S^{-1}A)^g$. In particular, $(A^g)_{\mathfrak{m}} = (A_{\mathfrak{m}})^g$ for any maximal ideal $\mathfrak{m}$ in A.*