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On Infinite-dimensional Algebras Satisfying the Maximal Condition for Subalgebras

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1.

It has been an open question whether there exists an infinite-dimensional Lie algebra satisfying the maximal condition for subalgebras. Recently in the paper [3], we have given an affirmative answer to this question by showing that the Lie algebra W introduced in [1, p. 177] is such a Lie algebra.

On the other hand, in the paper [2] R. K. Amayo has constructed a countable infinity of pair-wise non-isomorphic Lie algebras satisfying the maximal condition for subalgebras. The reasoning is however much complicated.

Thus in this paper we shall present simple and brief proofs of the results in [2] by reasoning along the same lines as in [3].

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2.

The fundamental tool which we employ is the lemma in [3]. We state it without proof in the following

LEMMA. *Let S be a subset of $\mathbf{N}$ satisfying the condition: If $s, t \in S$ and $s \neq t$, $s+t \in S$. Then there exists a finite number of different elements $s_1, s_2, \dots, s_r$ of S such that*

(i) *s_1 is the smallest element of S,*

(ii) $S=\{s_1\} \cup \{s_2+ns_1 | n=0, 1, 2,\dots\} \cup \cdots \cup \{s_r+ns_1 | n=0, 1, 2,\dots\}$.

Let $\mathfrak{k}$ be a field of characteristic 0. We let λ: $\mathbf{Z}\times\mathbf{Z}\to\mathfrak{k}$ be a map such that

$$(*) \qquad i \neq j \Rightarrow \lambda(i, j) \neq 0$$

and let $A(\lambda)$ be the infinite-dimensional (not necessarily associative) algebra over $\mathfrak{k}$ with basis $\{w(i) | i \in \mathbf{Z}\}$ and bilinear product defined by

$$w(i)\circ w(j) = \lambda(i, j)w(i+j), \qquad i, j \in \mathbf{Z}.$$

For any non-negative integer n, let $A(\lambda, n)$ be a subalgebra of $A(\lambda)$ generated by