

**Contributions to Balanced Fractional  $2^m$  Factorial  
Designs Derived from Balanced Arrays  
of Strength  $2l$**

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**0. Introduction and summary**

The theory of fractional factorial designs, first introduced by Finney [12], has found increasing use in agricultural, biological, industrial, and other various experimentations. One reason for the usefulness of fractional designs in preference to complete factorials is that they involve a lesser number of assemblies or treatment combinations, since higher order effects can be in general assumed negligible. In the beginning, the theory was developed for orthogonal fractional