HIROSHIMA MATH. J.
7 (1977), 201–215

On the Asymptotic Relationships between Two Systems of Differential Equations

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(Received September 10, 1976)

1. Introduction

In this paper we study the problem of asymptotic relationships between solutions of two systems of differential equations, one of which involves deviating arguments. We consider the systems

$$x'(t) = A(t)x(t) + f(t, x(g(t))), \tag{1}$$

$$y'(t) = A(t)y(t), \tag{2}$$

where $A(t)$ is a continuous $n \times n$ matrix function on $R_+ = [0, \infty)$, $f(t, z)$ is a continuous n-vector function on $R_+ \times R^n$, $g(t)$ is a continuous n-vector function on R_+ such that each component $g_i(t)$ is positive and satisfies $\lim_{t\to\infty} g_i(t) = \infty$, and

$$x(g(t)) = (x_1(g_1(t)), \dots, x_n(g_n(t))).$$

An important special case of (1) is the ordinary differential equation

$$x'(t) = A(t)x(t) + f(t, x(t)). \tag{3}$$

The problem of asymptotic relationships and/or asymptotic equivalence has been studied in many papers; see e.g. Brauer [1], Brauer and Wong [2], Cooke [3], Kato [5], Kitamura [6], Ráb [7], Švec [8], and the references cited in these papers. Recently, Ráb [7] and Kitamura [6] have presented conditions that lead to an equivalence between certain components of the solutions of (3) and the corresponding components of the solutions of (2).

The main purpose of this paper is to extend results of [6] to the systems (1) and (2) with general deviating argument $g(t)$ and to establish conditions that ensure the asymptotic equivalence of (1) and (2) when the deviating argument $g(t)$ is retarded.

In what follows we assume that the components $f_j(t, z)$ of $f(t, z)$ depend essentially on t and the q components $z_1, z_2, \dots, z_q$ $(1 \leqq q \leqq n)$ of z in the sense that

$$|f_j(t, z_1, \dots, z_n)| \leqq \omega_j(t, |z_1|, \dots, |z_q|) \tag{4}$$

for $(t, z) \in R_+ \times R^n$ and $j = 1, \dots, n$, where each $\omega_j(t, r_1, \dots, r_q)$ is continuous on