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(Z, λ)-Nuclear Mappings and Z-Nuclear Spaces

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Introduction

Persson and Pietsch [5] introduced the concepts of p-nuclear and p-quasi-nuclear mappings in Banach spaces. These concepts were recently extended in Miyazaki [4] to (p, q)-nuclear and (p, q)-quasi-nuclear mappings by using the sequence spaces $l_{p,q}$. On the other hand, these were extended in Ceĭtlin [1] to (Z, p)-nuclear and (Z, p)-quasi-nuclear mappings. The object of this paper is to extend these two kinds of concepts to (Z, λ)-nuclear and (Z, λ)-quasi-nuclear mappings in Banach spaces by making use of abstract sequence spaces λ. In case $1 \leq p \leq \infty$, $1 \leq q < \infty$, if $\lambda = l_{p,q}$ and Z is one-dimensional, a (Z, λ)-nuclear mapping coincides with a (p, q)-nuclear mapping, and if $\lambda = l_p$, a (Z, λ)-nuclear mapping coincides with a (Z, p)-nuclear mapping. We shall also extend the notion of nuclear spaces to Z-nuclear spaces by using (Z, l_1)-nuclear mappings introduced by Ceĭtlin [1]. We see that the tensor product of a nuclear space and a Banach space Z is Z-nuclear, and thus the space $\mathfrak{S}(R^n, Z)$ of rapidly decreasing functions defined in R^n and valued in Z is a Z-nuclear space.

In Section 1, we define the sequence space λ of type Λ and of type Λ_0 in such a way that $l_{p,q}$ is a space of type Λ_0 for $1 \leq p \leq \infty$, $1 \leq q \leq \infty$ and is a space of type Λ for $q \neq \infty$. In Section 2, we introduce the space $\lambda(Z)$ and consider the dual space of $\lambda(Z)$. Section 3 is devoted to studying (Z, λ)-nuclear mappings and Section 4 to studying (Z, λ)-quasi-nuclear mappings. We investigate Z-nuclear spaces in Section 5.

§1. Notations and Definitions

Let E and F be Banach spaces. We shall denote by $L(E, F)$ the space of continuous linear mappings T from E to F with the usual mapping norm

$$\|T\| = \sup_{\|u\| \leq 1} \|Tu\| .$$

We denote by $K(E, F)$ the space of compact mappings and by $L_0^Z(E, F)$ the space of mappings of Z-finite rank. Here $T \in L_0^Z(E, F)$ means that it can be written in the form