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On the Asymptotic Behavior of Nonoscillatory Solutions of Differential Equations with Deviating Arguments

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1. Introduction

Let r_i, $i=0, 1,\dots, n$ be positive continuous real-valued functions on the interval $[t_0, \infty)$. For a real-valued function h on $[T, \infty)$, $T \geqq t_0$, and any $k=0, 1,\dots, n$ we define the *k-th r-derivative* of h by the formula

$$D_r^{(k)}h = r_k(r_{k-1}(r_{k-2}(\cdots(r_1(r_0h)')'\cdots)')')'$$

when obviously we have

$$D_r^{(0)}h = r_0h$$

and

$$D_r^{(k)}h = r_k(D_r^{(k-1)}h)' \qquad (k = 1, 2,\dots, n).$$

Moreover, if $D_r^{(k)}h$ is defined as a continuous function on $[T, \infty)$, then h is said to be k-times *continuously* r-differentiable. We note that in the case where

$$r_0 = r_1 = \cdots = r_n = 1$$

the above notion of r-differentiability specializes to the usual one.

Now, we consider the n-th order $(n>1)$ differential equation with deviating arguments of the form

$$(E_m) \qquad (D_r^{(n)}x)(t) + a(t)F(x[\sigma_1(t)],\dots, x[\sigma_m(t)]) = b(t), \qquad t \geqq t_0,$$

where $r_n=1$. The continuity of the functions involved in the above equation (E_m) as well as sufficient smoothness to guarantee the existence of solutions of (E_m) on an infinite subinterval of $[t_0, \infty)$ will be assumed without mention. In what follows the term "solution" is always used only for such solutions $x(t)$ of (E_m) which are defined for all large t. The oscillatory character is considered in the usual sense, i.e. a continuous real-valued function which is defined on an interval of the form $[T, \infty)$ is called *oscillatory* if it has no last zero, and otherwise it is called *nonoscillatory*.

Furthermore, the conditions (i) and (ii) below are assumed to hold through-