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On Distributions Measured by the Riemann-Liouville Operators Associated with Homogeneous Convex Cones

Toshihiro WATANABE

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Introduction

This article deals with Riemann-Liouville operators associated with homogeneous convex cones V which have been studied by M. Riesz [5], L. Gårding [2] and S. G. Gindikin [3], and sets up a theory of distributions measured by the operators $\mathscr{P}^{\varrho}_{V\pm}$ (as for the definition, see (1–5) of § 1).

È. B. Vinberg, S. G. Gindikin, and I. I. Pyateckiĭ-Šapiro [9] have proved that every complex bounded homogeneous domain is analytically equivalent to an affine-homogeneous Siegel domain of the first or second kind, and then it is easy to prove that any affine homogeneous real domain is affine-equivalent to a convex linear homogeneous cone or a real Siegel domain (cf. [3], [8]).

We shall define a homogeneous distribution associated with a homogeneous domain D. Let G be a group to act transitively on D. Then G operates on the Schwartz space defined on D such that

$$(0\text{–}1) \qquad (f, g) \longmapsto f^g \quad \text{defined by} \quad f^g(x) = f(gx),$$

and by the duality, on the distribution space such that

$$(0\text{–}2) \qquad (g, \Delta) \longmapsto g\Delta \quad \text{defined by} \quad (g\Delta)(f) = \Delta(f^g).$$

Let ω be a one-dimensional representation on G. If the distribution Δ satisfies the relation

$$(0\text{–}3) \qquad g\Delta = \omega(g)^{-1}\Delta$$

for any $g \in G$, it is called a homogeneous distribution associated with D (cf. [10]). Then the homogeneous distribution Δ is extended to the whole space such that Δ_+ is equal to Δ on D, and to 0 for other else. If a set M of homogeneous distributions depends on a parameter, and for any Δ_α, Δ_β in M the convolution operator $(\Delta_\alpha)_+ * (\Delta_\beta)_+$ is well defined and equals $(\Delta_{\alpha+\beta})_+$ in M, an operator $(\Delta_{\alpha+} f) \longmapsto \Delta_{\alpha+} * f$ is called the Riemann-Liouville operator associated with the domain D. Therefore the operator $\mathscr{P}^{\varrho}_{V+}$ is one of canonical Riemann-Liouville operators, and satisfies the Huygens principle (cf. [7]).