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Propagation of Chaos for Boltzmann-like Equation of Non-cutoff Type in the Plane

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§1. Introduction

Consider a rarefied monoatomic gas composed of a large number of (say, N) molecules moving in space according to the law of classical mechanics and colliding in pairs from time to time. Assume that the motion is specified by giving the intermolecular forces, which are supposed to be given by pair forces only. Let $Nu(t, x)dx$ be the number of molecules with velocities belonging to dx at time t. Then the time evolution of the density $u(t, x)$ in the spatially homogeneous case is given by the following Boltzmann equation:

$$(1.1)\qquad \frac{\partial u(t, x)}{\partial t} = \int_{(0,\pi)\times(0,2\pi)\times R^3} \{u(t, x')u(t, y') - u(t, x)u(t, y)\} \times |x-y| I(|x-y|, \theta) \sin\theta d\theta d\epsilon\, dy,$$

where x' and y' stand for the velocities after collision of molecules with velocities x and y, I is the differential scattering cross section and $\theta\,(\epsilon)$ is the collatitude (the longitude) which measures the scattering angle formed by $y-x$ and $y'-x'$.

When the pair forces are determined by the power-law potential proportional to ρ^{-4} (Maxwellian molecules), $|x-y|I(|x-y|, \theta)$ becomes a function of θ alone; here ρ is the distance of the colliding molecules. For these materials, see Uhlenbeck and Ford [17]. In this case Tanaka [14] constructed the associated Markov process by making use of the stochastic integral equation based upon a Poisson random measure.

The purpose of this paper is to prove the propagation of chaos for two-dimensional analogous model of non-cutoff type by using similar stochastic integral equations.

Propagation of chaos was first discovered by Kac [7] for a model of the Maxwellian gas of cutoff type. The statement of propagation of chaos for (1.1) in the sense of Kac is that if $u_n = u_n(t, x_1, \dots, x_n)$ is the solution of the forward equation of n-molecules

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