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A Note on Witt Algebras

Fujio KUBO

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Introduction

Let $\mathfrak{k}$ be a field and let G be a subgroup of the additive group $\mathfrak{k}^+$. Let $W(G)$ be a Lie algebra with basis $\{w(g) | g \in G\}$ and multiplication

$$[w(g), w(h)] = (g - h)w(g + h), \qquad g, h \in G.$$

This Lie algebra $W(G)$ is called the Witt algebra for G. These Lie algebras were studied by R. K. Amayo and I. Stewart in their book [1, pp. 206–212]. Theorem 3.4 in [1, p. 208] states as follows.

THEOREM A. *If $\mathfrak{k}$ has characteristic* 0 *and $0 \neq G \leqq \mathfrak{k}^+$, then the finite-dimensional subalgebras of $W(G)$ are precisely* (a) *the* 1*-dimensional subalgebras $<x>$ for $x \in W(G)$ and* (b) *the subalgebras of* 3*-dimensional algebras spanned by $\{w(-g), w(0), w(g)\}$ for $0 \neq g \in G$.*

In [1, p. 210] they made a

CONJECTURE. *Residually nilpotent Lie algebras satisfying the maximal condition for subideals may have no infinite-dimensional abelian subalgebras.*

In this paper, we shall point out that Theorem A is not correct, and establish the correct form of Theorem A. We shall also give an example to illustrate that the above conjecture is not true.

1. Notations and preliminary results

Let $\mathfrak{k}$ be of characteristic 0 and G be a fixed subgroup of $\mathfrak{k}^+$ with $|G| > 1$. Any torsion-free abelian group can be linearly ordered (Neumann [2]), so we may equip G with a linear ordering $<$. For any element

$$0 \neq x = \Sigma x_g w(g)$$

of $W(G)$, where $x_g \in \mathfrak{k}$ and the summation is over a finite number of $g \in G$, we define

$$\max(x) = \max\{g | x_g \neq 0\}, \quad \min(x) = \min\{g | x_g \neq 0\},$$

and for any subset $S \neq 0$ of $W(G)$ we define