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Meromorphic Mappings into a Compact Complex Space

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Introduction. Let M be an m-dimensional smooth complex projective variety, $\Delta_n=\{(z_j)\in \boldsymbol{C}^n; |z_j|<1\}$ the unit polydisc in the complex affine space $\boldsymbol{C}^n$ of dimension n and $\Delta_n^*=\Delta_n-\{z_1=0\}$. Kobayashi-Ochiai ([5]) proved that if a holomorphic mapping $f: \Delta_m^*\to M$ is of rank m, i.e., the differential df is non-singular at some point, and if the canonical bundle K_M over M is positive, then f has a meromorphic extension from Δ_m into M. Kodaira showed that this extension theorem remains valid in the case where M is of general type (see Kobayashi-Ochiai [5, Addendum]). The condition that M is of general type is birationally invariant, whereas the positivity of K_M is not. For holomorphic mappings $f: \Delta_n^*\to M$ with $n<m$, Carlson ([1]) proved the analogous extension theorem under the condition that the vector bundle $\Omega(n)$ of holomorphic n-forms over M is positive.

In the present paper we shall establish such an extension theorem for algebraically non-degenerate holomorphic mappings $f: \Delta_n^*\to M$ with $n\leqq m$ under an assumption for $\Omega(n)$ which is birationally, moreover, bimeromorphically invariant and coincides, in case $n=m$, with that M is of general type (see (1.1), Corollary 1.2 and Theorem 3.1). Furthermore we shall deal with the case where M is a Moišezon space.

In the proof of that theorem, the key is a lemma of the Schwarz type (Lemma 2.2). In the last section we shall apply this lemma to study the family $\mathscr{M}$ of meromorphic mappings from an n-dimensional compact complex manifold N into M of rank n. We shall prove that if the analytic set B in M defined in section 4 is empty, then $\mathscr{M}$ is m-normal and the limits belong to $\mathscr{M}$, i.e., the space $\mathscr{M}$ endowed with m-convergence is compact (see Definitions 5.1, 5.2 and Theorem 5.3). In general, however, it seems that the m-convergence does not determine a topology in the precise sense. To make clear this fact we shall prove that $\mathscr{M}$ can be embedded into some complex affine and projective spaces in such a way that $\mathscr{M}$ is compact in each of them (Theorem 5.4).

1. Preliminaries

Let M be a compact complex manifold of dimension m and $\Omega(n)$ the vector bundle of holomorphic n-forms over M. Suppose that there exists an effective