HIROSHIMA MATH. J.
7 (1977), 827–832

On Removable Singularities for Polyharmonic Distributions

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(Received May 19, 1977)

1. Introduction

Throughout this paper let $1<p<\infty$, $1/p+1/q=1$ and m be a positive integer. For an open set G in the n-dimensional Euclidean space R^n, we denote by $BL_m(L^q(G))$ the space of all distributions on G whose distributional derivatives of order m are all in $L^q(G)$, that is, a distribution T on G belongs to $BL_m(L^q(G))$ if and only if

$$|T|_{m,q}=|T|_{m,q,G}=(\sum_{|\alpha|=m}\|D^\alpha T\|^q_{L^q(G)})^{1/q}<\infty,$$

where α is an n-tuple $(\alpha_1, \alpha_2,\dots, \alpha_n)$ of non-negative integers with length $|\alpha|=\alpha_1+\alpha_2+\cdots+\alpha_n$, $D^\alpha=\partial^{|\alpha|}/\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\cdots\partial x_n^{\alpha_n}$ and $\|\cdot\|_{L_q(G)}$ denotes the L^q-norm on G. We write simply $\|\cdot\|_q$ for $\|\cdot\|_{L^q(R^n)}$. We denote by Δ^m the Laplace operator iterated m times and write simply Δ for Δ^1. The value of a distribution T on G at $\varphi\in C_0^\infty(G)$ is denoted by $<T, \varphi>$.

Let E be a compact set in R^n. L. I. Hedberg proved the following result ([5; Theorem 1]): Let $\mathscr{C}$ be the space $C_0^\infty(R^n\backslash E)$ or the space of all functions $\varphi\in C_0^\infty(R^n)$ such that $|\mathrm{grad}\,\varphi|=0$ on a neighborhood of E. Then $\mathscr{C}$ is dense in $C_0^\infty(R^n)$ with respect to the norm $|\cdot|_{1,p}$ if and only if any $T\in BL_1(L^q(R^n))$ such that $<T, \Delta\varphi>=0$ for any $\varphi\in\mathscr{C}$ is harmonic on R^n. We generalize this result as follows:

THEOREM 1. *Let $\mathscr{C}$ and $\mathscr{C}'$ be subspaces of $C_0^\infty(R^n)$ such that $\mathscr{C}\subset\mathscr{C}'$. Then $\mathscr{C}$ is dense in $\mathscr{C}'$ with respect to the norm $|\cdot|_{m,p}$ if and only if any $T\in BL_m(L^q(R^n))$ such that $<T, \Delta^m\varphi>=0$ for any $\varphi\in\mathscr{C}$ satisfies $<T, \Delta^m\psi>=0$ for any $\psi\in\mathscr{C}'$.*

As an application of this theorem, we shall give a condition, in terms of capacity, for a compact set in R^n to be removable for a class of polyharmonic distributions.

2. Proof of Theorem 1

We first suppose that $\mathscr{C}$ is dense in $\mathscr{C}'$ with respect to $|\cdot|_{m,p}$. We write