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Extremal Length of an Infinite Network Which is not Necessarily Locally Finite

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Introduction

In the preceding paper [2], we introduced a generalized extremal length of an infinite network N which is locally finite, i.e., each node has only a finite number of incident arcs, and investigated the generalized reciprocal relation between the extremal distance $EL_p(A, B)$ (resp. $EL_p(A, \infty)$) and the extremal width $EW_q(A, B)$ (resp. $EW_q(A, \infty)$) relative to mutually disjoint nonempty finite subsets A and B of nodes (resp. a finite subset A of nodes and the ideal boundary ∞ of the network N). In this paper we shall be concerned with the same problem on an infinite network which is not necessarily locally finite. It will be shown in §2 that the generalized reciprocal relation between $EL_p(A, B)$ and $EW_q(A, B)$ still holds in the case where N is not necessarily locally finite. However, the generalized reciprocal relation between $EL_p(A, \infty)$ and $EW_q(A, \infty)$ does not hold, in general, in the present case. In §3 we shall introduce a p-almost locally finite network, for which the generalized reciprocal relation holds. We shall also study the stability of $\{EL_p(A, X-X_n)\}$ and $\{EW_q(A, X-X_n)\}$ with respect to an exhaustion $\{<X_n, Y_n>\}$ of N in the case where N is a p-almost locally finite network.

§1. Preliminaries

Let X be a finite or countably infinite set of nodes, let Y be a finite or countably infinite set of arcs and let K be a function on $X \times Y$ satisfying the following conditions:

(1.1) The range of K is $\{-1, 0, 1\}$.

(1.2) For each $y \in Y$, $e(y) \equiv \{x \in X; K(x, y) \neq 0\}$ consists of exactly two nodes x_1, x_2 and $K(x_1, y)K(x_2, y) = -1$.

(1.3) For any $x, x' \in X$, there are $x_1, \dots, x_n \in X$ and $y_1, \dots, y_{n+1} \in Y$ such that $e(y_j) = \{x_{j-1}, x_j\}$, $j = 1, \dots, n+1$ with $x_0 = x$ and $x_{n+1} = x'$.

For each $x \in X$, the set

$$Y(x) = \{y \in Y; K(x, y) \neq 0\}$$