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On Weierstrass Points of Non-hyperelliptic Compact Riemann Surfaces of Genus Three

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The purpose of this paper is, first, to determine the equations of compact Riemann surfaces of genus three, considering these surfaces as coverings of the Riemann sphere. We obtain Theorem 1 which asserts that the equations are given by

$$y^3 - \gamma_2(x)y - \gamma_3(x) = 0.$$

Here $\gamma_2(x)$ is a polynomial of degree 3 or less than 3, and $\gamma_3(x)$ is a polynomial of degree 5 or 4. Both of them depend on Weierstrass points.

Next, we construct a basis of differentials of the first kind for these Riemann surfaces. Using these results, we investigate Weierstrass points of these Riemann surfaces. Our main interest is to determine Riemann surfaces which have exactly 12 Weierstrass points. The number 12 is the smallest one for all the non-hyperelliptic compact Riemann surfaces of genus three. We obtain Theorem 2 which asserts that Riemann surfaces having just 12 Weierstrass points are exactly two and these equations in homogeneous coordinates are given by

(1) $$x^4 + y^4 + z^4 = 0$$

and

(2) $$x^4 + y^4 + z^4 + 3(x^2y^2 + y^2z^2 + z^2x^2) = 0.$$

§1. Preliminaries

Given any point P on a compact Riemann surface of genus g (≥ 1), there are exactly g orders which can be specified

$$1 = n_1 < n_2 < \cdots < n_3 < 2g$$

such that there does not exist any meromorphic function on the surface whose only singularity is a pole of order n_i ($1 \leq i \leq g$) at P.

These g orders are called the gaps at P. A point whose gap sequence contains an integer greater than g is called a Weierstrass point.