HIROSHIMA MATH. J.
7 (1977), 707–730

Boundary Value Control Theory of Elastodynamic System

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(Received May 11, 1977)

1. Introduction

Consider a linear elastic solid occupying, in its non deformed state, a bounded three dimensional domain Ω with a C^∞-boundary $\partial\Omega$. Let the medium be fixed on some part of the boundary and free on the other part. In this paper we consider the problem of controlling the deformation of the medium by applying traction forces on a small subset of the free boundary part. Let us denote by $\{u^i(x, t)\}_{i=1,2,3}$ the displacement vector at the time t of the material particle which lies at $x=\{x_i\}_{i=1,2,3}$ in the non deformed state. Then $u^i(x, t)$ $(i=1, 2, 3)$ satisfy the system of equations

$$(1.1) \qquad \rho(x)\frac{\partial^2 u^i}{\partial t^2} - \frac{\partial}{\partial x_j}\left(c_{ijkl}(x)\frac{\partial u^k}{\partial x_l}\right) = 0 \qquad \text{in} \quad Q \equiv \Omega \times (0, T)$$

with initial conditions

$$(1.2) \qquad u^i(x, 0) = 0 \qquad \text{in} \quad \Omega,$$

$$(1.3) \qquad [\partial u^i/\partial t](x, 0) = 0 \qquad \text{in} \quad \Omega$$

and mixed boundary conditions

$$(1.4) \qquad u^i(x, t) = 0 \qquad \text{on} \quad \Gamma_1 \times (0, T),$$

$$(1.5) \qquad n_j c_{ijkl}(x)\frac{\partial u^k}{\partial x_l}(x, t) = g^i(x, t) \qquad \text{on} \quad \Gamma_2 \times (0, T).$$

Here $n=(n_1, n_2, n_3)$ is the outward unit normal vector on $\partial\Omega$, Γ_1 and Γ_2 are disjoint relatively open subsets of $\partial\Omega$ such that $\partial\Omega=\bar{\Gamma}_1 \cup \Gamma_2=\Gamma_1 \cup \bar{\Gamma}_2$, $L=\bar{\Gamma}_1 \cap \bar{\Gamma}_2$ is a smooth curve and T is a positive number. The coefficients $\rho(x)$ and $c_{ijkl}(x)$ are assumed to be C^∞-functions and to satisfy the following symmetry and definiteness conditions:

$$\rho_m^2 \leqq \rho(x) \leqq \rho_M^2 \qquad \text{in} \quad \Omega, \qquad 0 < \rho_m \leqq \rho_M,$$

$$|(\partial\rho/\partial x_i)(x)| \leqq \rho_0^2 \qquad \text{in} \quad \Omega, \qquad i = 1, 2, 3,$$

$$c_{ijkl}(x) = c_{klij}(x) \qquad \text{in} \quad \Omega,$$