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Nonoscillation Criteria for Differential Equations of the Second Order

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1. Introduction

In this paper we consider the perturbed second order nonlinear differential equation

$$(E) \qquad (a(t)x')' + q(t)f(x) = e(t, x, x').$$

In the last twenty years many authors have studied the oscillatory behavior of equations of this type especially when $e(t, x, x') \equiv 0$. Fortunately, several surveys of known results have been done, the most recent of which are by Wong [8, 9]. While many sufficient conditions for oscillation are known, there are relatively few theorems which guarantee that (E) has a nonoscillatory solution (see [1–9] and the references contained therein). Far fewer results guaranteeing that all solutions of (E) be nonoscillatory are known, and in fact, when $e(t, x, x') \not\equiv 0$, only the results of Graef [1] and Graef and Spikes [2–7] apply.

In this paper we obtain sufficient conditions for all solutions of (E) to be nonoscillatory. This is accomplished by comparing (E) to an unperturbed nonlinear equation in Theorems 3 and 4 and to an unperturbed linear equation in Theorem 5. Use is made of a nonlinear Picone type identity introduced by the authors in [7].

2. Nonoscillation Criteria

Consider the equations

$$(1) \qquad (a(t)x')' + q(t)f(x) = e(t, x, x')$$

and

$$(2) \qquad (a_1(t)x')' + q_1(t)f_1(x) = 0,$$

where $a, a_1, q, q_1: [t_0, \infty) \to R, f, f_1: R \to R$, and $e: [t_0, \infty) \times R^2 \to R$ are continuous, $a(t) > 0$, and $a_1(t) > 0$. It will be convenient to use the same classifica-

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