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On Claw-Decomposition of a Complete Multi-Partite Graph[*)]

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1. Introduction

A multi-partite graph, denoted by $G_m(n_1, n_2,..., n_m)$, is a graph whose point set can be partitioned into m subsets $V_1, V_2,..., V_m$ with $n_1, n_2,..., n_m$ points each, such that every line of $G_m(n_1, n_2,..., n_m)$ joins different subsets. If $G_m(n_1, n_2,..., n_m)$ contains every line joining different subsets, then it is called a complete m-partite graph and is denoted by $K_m(n_1, n_2,..., n_m)$. A complete bipartite graph $K_2(1, c)$ is called a *claw* or a c-claw by specifically indicating its number of lines.

A graph G is said to be c-claw decomposable if it can be decomposed into pairwise line-disjoint claws with c lines each.

The problem of c-claw decomposability of the complete graph and that of the complete bigraph have been raised and cleared up by Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [2]. The purpose of this paper is to give an analogous claw-decomposition theorem for a complete m-partite graph $K_m(n, n,..., n)$ with m sets of n points each.

2. A claw-decomposition theorem

With respect to the claw-decomposition of a complete m-partite graph $K_m(n, n,..., n)$, we have the following theorem.

THEOREM. *$K_m(n, n,..., n)$ can be decomposed into pairwise line-disjoint c-claws if and only if* (i) $\binom{m}{2}n^2 \equiv 0 \pmod{c}$ *and* (ii) $mn \geq 2c$.

Since $K_m(n, n,..., n)$ turns out to be the complete graph K_m when $n=1$, our theorem is a generalization of the c-claw decomposition theorem for K_m which has been given in [2] and has been applied to the design of an efficient storage and retrieval system in [3]. This is also a partial generalization of the theorem for a complete bipartite graph $K_2(n_1, n_2)$ given in [2].

Suppose an arbitrary direction of adjacency is assigned on every line of

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