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The Plancherel Formula for a Pseudo-Riemannian Symmetric Space

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§1. Introduction

The Plancherel theorem for Riemannian symmetric spaces has been extensively studied and almost completely established by several authors.

Now we believe that it is interesting and important to verify the analogous theorem for pseudo-Riemannian symmetric spaces. (For the definition of a pseudo-Riemannian symmetric space, see [7]). There have been several results in this direction; see [2], [3], [4], [10], [12], [13], [14], [15] and [16].

In this paper we prove the Plancherel formula for the pseudo-Riemannian symmetric space $SU(p, 1)/S(U(1)\times U(p-1, 1))$. Our method, which is mainly due to the theory of Kokaira and Titchmarsh, may be applicable to more general pseudo-Riemannian symmetric spaces.

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2. The main result

Let the form $[z, w]=z_1\bar{w}_1+\cdots+z_p\bar{w}_p-z_{p+1}\bar{w}_{p+1}$ be given in the complex $p+1$-dimensional space $\boldsymbol{C}^{p+1}$ $(p\geqq 2)$; let G be the linear group of transformations which have determinant 1 and leave this form invariant.

The mapping

$$\sigma\colon g \longrightarrow J({}^t\bar{g})^{-1}J$$

is an involutive automorphism of G, where $J=\mathrm{diag}(-1, 1,\dots, 1, -1)$. The fixed points of σ constitute the subgroup

$$H=\left\{\begin{pmatrix} e^{i\theta} & 0\cdots 0 \\ 0 & \\ \vdots & * \\ 0 & \end{pmatrix}\in G;\ \theta\in\boldsymbol{R}\right\}.$$

Furthermore there exists a G-invariant indefinite Riemannian metric on G/H and, therefore, G/H is a pseudo-Riemannian symmetric space.