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On the Trace Mappings for the Space $B_{p,\mu}(R^N)$

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1. Introduction

By $H^{\mu}(R^N)$ we shall understand the space of $u \in \mathscr{S}'(R^N)$ such that its Fourier transform $\hat{u}$ is a locally summable function satisfying

$$\|u\|_{\mu}^2 = \left(\frac{1}{2\pi}\right)^N \int_{\Xi^N} |\hat{u}(\xi)|^2 \mu^2(\xi) d\xi < \infty,$$

where R^N is an N-dimensional Euclidean space, Ξ^N its dual Euclidean space and μ is a temperate weight function in Ξ^N. In our previous paper [2] we have given a trace theorem for the space $H^{\mu}(R^N)$. Let $\mu = \mu(\xi', \tau)$, $\xi' = (\xi_1, \ldots, \xi_n)$, $\tau = (\tau_1, \ldots, \tau_m)$, $N = n + m$ and assume $\int_{\Xi^m} \frac{|\tau|^{2M}}{\mu^2(\xi', \tau)} d\tau < \infty$ for a non-negative integer M. Put $\nu_k(\xi') = \left\{\int_{\Xi^m} \frac{\tau^{2k}}{\mu^2(\xi', \tau)} d\tau\right\}^{-1/2}$ for $k = (k_1, \ldots, k_m)$, k_j being a non-negative integer, such that $|k| \leqq M$. Then the mapping

$$H^{\mu}(R^N) \ni u \longrightarrow \{D_t^k u(x', 0)\} \in \prod_{|k| \leq M} H^{\nu_k}(R^n)$$

is an epimorphism if and only if there exists a positive constant C such that $\det|\kappa_{k+l}| \geqq C \prod_{|k| \leq M} \kappa_{2k}$ with $\kappa_k(\xi') = \int_{\Xi^m} \frac{\tau^k}{\mu^2(\xi', \tau)} d\tau$.

The purpose of this paper is to investigate the trace mappings for the space $B_{p,\mu}(R^N)$, $1 < p < \infty$, which consists of all distributions $u \in \mathscr{S}'(R^N)$ such that $\hat{u}$ is a function and

$$\|u\|_{p,\mu}^p = \left(\frac{1}{2\pi}\right)^N \int_{\Xi^N} |\hat{u}(\xi)|^p \mu^p(\xi) d\xi < \infty.$$

Here $B_{2,\mu}(R^N) = H^{\mu}(R^N)$. We shall give some sufficient conditions for the trace mapping of above type for $B_{p,\mu}(R^N)$ to be an epimorphism. We shall also investigate the trace mappings by making a comparison with the notions of multiplication of distributions and section of distributions.

2. Preliminaries

We shall use the same notations and terminologies as in our previous paper