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Note on KO-Rings of Lens Spaces Mod 2^r

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§1. Introduction

Let η be the canonical complex line bundle over the standard lens space mod p^r:

$$L^n(p^r) = S^{2n+1}/Z_{p^r} \qquad (p\text{: prime, } r \geqq 1;\ n \geqq 0).$$

Then, we have the stable classes

$$(1.1) \qquad \sigma = \eta - 1 \in \tilde{K}(L^n(p^r)), \quad r\sigma = r\eta - 2 \in \widetilde{KO}(L^n(p^r)),$$

where r is the real restriction. On the orders of the powers of these elements, the following results are proved in [1, Th. 1.1]:

(1.2) *$\sigma^i \in \tilde{K}(L^n(p^r))$ is of order $p^{r+[(n-i)/(p-1)]}$ for $1 \leqq i \leqq n$, and $\sigma^{n+1} = 0$.*

(1.3) *If p is an odd prime, then $(r\sigma)^i \in \widetilde{KO}(L^n(p^r))$ is of order $p^{r+[(n-2i)/(p-1)]}$ for $1 \leqq i \leqq [n/2]$, and $(r\sigma)^{[n/2]+1} = 0$.*

The purpose of this note is to prove the following theorem, by using the partial result of M. Yasuo [5, Prop. (3.5)] which shows the theorem under the assumption $n \not\equiv 1\,(4)$:

THEOREM 1.4. *In the reduced KO-group $\widetilde{KO}(L^n(2^r))$ $(r \geqq 2)$, the order of $(r\sigma)^i$ is equal to*

$$2^{r+n-2i+1} \text{ if } n \equiv 0\,(2), \quad 2^{r+n-2i} \text{ if } n \equiv 1\,(2), \qquad \text{for } 1 \leqq i \leqq [n/2];$$

$$1 \text{ if } n \not\equiv 1\,(4), \quad 2 \text{ if } n \equiv 1\,(4), \qquad \text{for } i = [n/2]+1;$$

and 1 *for* $i \geqq [n/2]+2$.

As an application of this theorem, we have the following corollary by the method of M. F. Atiyah using the γ-operation.

COROLLARY 1.5 (cf. [3, Th. C, Prop. 7.6]). *The $(2n+1)$-manifold $L^n(2^r)$ $(r \geqq 2)$ cannot be immersed in the Euclidean space R^{2n+2L} and cannot be imbedded in $R^{2n+2L+1}$, where*