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On the Existence of Non-tangential Limits of Polyharmonic Functions

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1. Introduction and statement of results

Let R^n ($n \geqq 2$) be the n-dimensional Euclidean space. A point x of R^n will be written also as $(x', x_n) \in R^{n-1} \times R^1$. We denote by R^n_+ the set of all points $x=(x', x_n) \in R^n$ such that $x_n > 0$, and by R^n_0 its boundary ∂R^n_+. For a function $u \in C^\infty(R^n_+)$, we define the gradient of order k by

$$\nabla^k u(x) = (D^\gamma u(x))_{|\gamma|=k}, \qquad x \in R^n_+,$$

where $\gamma=(\gamma_1,\dots,\gamma_n)$ is a multi-index with length $|\gamma| = \sum_{i=1}^n \gamma_i$ and $D^\gamma = (\partial/\partial x_1)^{\gamma_1} \cdots (\partial/\partial x_n)^{\gamma_n}$. A function $u \in C^\infty(R^n_+)$ is said to be polyharmonic of order m in R^n_+ if $\Delta^m u = 0$ on R^n_+, and to have a non-tangential limit at $\xi \in R^n_0$ if

$$\lim_{\substack{x \to \xi \\ x \in \Gamma(\xi; a)}} u(x)$$

exists and is finite for all $a > 0$, where Δ^m is the Laplace operator iterated m times and

$$\Gamma(\xi; a) = \{x = (x', x_n) \in R^n_+;\ |(x', 0) - \xi| < a x_n,\ |x - \xi| \leqq 1\}.$$

Our first aim is to show the following theorem:

THEOREM 1. *Let k and m be positive integers such that $k \geqq m$, $1 < p < \infty$ and $-\infty < \alpha < kp$. If u is a function polyharmonic of order m in R^n_+ which satisfies*

$$\iint_G |\nabla^k u(x', x_n)|^p x_n^\alpha dx' dx_n < \infty \qquad \text{for any bounded open set } G \subset R^n_+,$$

then there exists a Borel set $E \subset R^n_0$ such that $B_{k-\alpha/p,p}(E) = 0$ and u has a non-tangential limit at each point of $R^n_0 - E$.

Here $B_{\beta,p}$ ($\beta > 0$) is the Bessel capacity of index (β, p) (cf. [2]). Theorem 1 is a generalization of a result of the first author [3; Theorem 1] ($k=m=1$). In case $-1 < \alpha < kp-1$, Theorem 1 is the best possible as to the size of the exceptional set in the following sense: