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A Correction to "Forced Oscillations in General Ordinary Differential Equations with Deviating Arguments"

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1. Introduction

In [1] this author presented conditions to ensure that all oscillatory solutions of the equation

$$(1)\qquad (r(t)y'(t))^{(n-1)} + a(t)y_\tau(t) = f(t), \quad y_\tau(t) \equiv y(t-\tau(t))$$

approach zero as $t \to \infty$. The proof of the main result (Lemma (1)) was based on the "truth" of the inequality

$$(2)\qquad \left|\int_{t_1}^{t_2}\int_s^p a(t)dt\,ds\right| \le \int_{t_1}^{t_2}\int_t^{t_2} |a(s)|ds\,dt$$

where $t_1 < p < t_2$ and $a(t)$ continuous in $[t_1, t_2]$.
But this inequality (cf. Staikos and Philos [2]) is false as the following counter example (due to Prof. T. Kusano of Hiroshima University) shows:

$$\int_\pi^{5\pi}\int_s^{5\pi} |f(t)|dt\,ds = 3\pi \quad\text{and}\quad \left|\int_\pi^{5\pi}\int_s^{2\pi} f(t)dt\,ds\right| = 5\pi$$

where

$$f(t) = \begin{cases} 0 & (\pi \le t < 2\pi) \\ \sin t & (2\pi \le t \le 3\pi) \\ 0 & (3\pi \le t \le 5\pi). \end{cases}$$

However the conclusion of this crucial lemma remains true with a very minor change. We shall consider the following more general equation

$$(3)\qquad (r(t)y'(t))^{(n-1)} + a(t)h(y(g(t))) = f(t)$$

subject to similar assumptions. More precisely we assume

(i) $a(t), r(t), g(t), h(t), f(t)$ are real, continuous on the whole real line R:

(ii) $r(t) > 0$, $g(t) \le t$, $g(t) \to \infty$ as $t \to \infty$;

(iii) $0 \le \dfrac{h(t)}{t} \le m$, for some $m > 0$, $t > 0$.