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On a posteriori Error Estimation in the Numerical Solution of Systems of Ordinary Differential Equations

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1. Introduction

Consider the boundary value problem

$$\frac{dx}{dt} = X(x, t), \qquad a \leqq t \leqq b, \tag{1.1}$$

$$f[x] = 0, \tag{1.2}$$

where x and $X(x, t)$ are real n-vectors, f is an operator from $D \subset C[J]$ into R^n which is continuously Fréchet differentiable in D, and $C[J]$ is the space of n-vector functions continuous on $J=[a, b]$.

In our previous paper [2], replacing (1.1) with an equivalent system of integral equations, we obtained *a posteriori* error estimates of continuous approximate solutions of (1.1), (1.2). In those estimates the fundamental matrix of a linear homogeneous system of differential equations plays an important role and its inverse matrix is also required. In many practical applications, however, exact fundamental matrices and their exact inverses are not available, so that the estimates are not always applicable.

The object of this paper is to give error estimates of approximate solutions in terms of approximate fundamental matrices and their approximate inverses. In Section 3 error estimates are obtained in the case where approximate fundamental matrices are continuous and also in the case where they are continuously differentiable. The results are illustrated with a numerical example.

In Section 4 we treat the case where the boundary condition depends on the fundamental matrices of the first variation equation of (1.1).

2. Notations and preliminaries

Let R^n denote a real n-space with any norm $\|\cdot\|$ and let $C[J]$ be the Banach space of all real n-vector functions $x(t)$ continuous on the interval $J=[a, b]$ with the norm $\|x\|_c = \sup_{t \in J} \|x(t)\|$. For any fixed $t_0 \in J$ let

$$C_0[J] = \{x \in C[J] \mid x(t_0) = 0\}.$$