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Extension Theorems on Some Generalized Nilpotent Lie Algebras

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1. Introduction

Let L be a Lie algebra over a field. It is well known that if I is a nilpotent ideal of L and L/I^2 is nilpotent, then L is nilpotent. In group theory a theorem asserting that for a normal nilpotent subgroup N of a group G a property of G/N' passes to G is termed one of Hall's type, and it has been shown for several properties including the nilpotency ([5], [6, p. 57]). In connection with these, it seems interesting for us to investigate the Lie-theoretic analogue of a theorem of Hall's type. The aim of this paper is to show the following extension theorem in Lie algebras: Let $\mathfrak{X}$ be one of the classes $\mathfrak{Z}$, $\mathfrak{E}$, $\mathrm{L}\mathfrak{N}$, $\mathfrak{F}\mathrm{t}$, $\mathfrak{B}$, $\mathfrak{G}\mathrm{r}$ of Lie algebras. If I is a nilpotent ideal of L and L/I^2 lies in $\mathfrak{X}$, then L lies in $\mathfrak{X}$.

2. Notations

Throughout this paper we consider Lie algebras over an arbitrary field Φ which are not necessarily finite-dimensional.

Let L be a Lie algebra and H be a subalgebra of L. We use the following notations as usual.

H si L: H is a subideal of L.

H asc L: H is an ascendant subalgebra of L, i.e., there exists an ascending series $\{H_\beta: 0\leqq\beta\leqq\alpha\}$ of subalgebras of L, indexed by ordinals $\beta\leqq\alpha$, such that $H_0=H$, $H_\alpha=L$, $H_\beta \lhd H_{\beta+1}$ for all $\beta<\alpha$, and $H_\lambda=\bigcup_{\beta<\lambda} H_\beta$ for all limit ordinals $\lambda\leqq\alpha$.

$\zeta_\alpha(L)$: the α-th term of the upper central series of L (α: an ordinal). In particular $\zeta_1(L)$ is the center of L.

$\zeta_*(L)$: the hypercenter of L.

We say that $x\in L$ is a right Engel element if for each $y\in L$ there exists a non-negative integer $n=n(x, y)$ such that $[x, {}_ny]=0$.

$\mathfrak{r}(L)$: the set of right Engel elements of L.

Let us recall several classes of Lie algebras.

$\mathfrak{N}$: the class of nilpotent Lie algebras.

$\mathfrak{Z}$: the class of hypercentral Lie algebras.