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The Steenrod Operations in the Eilenberg-Moore Spectral Sequence

Masamitsu MORI

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Introduction

R. Vázquez García [19] and S. Araki [1] introduced two kinds of the Steenrod operations into the mod p Serre spectral sequence $\{E_r\}$, that is, the squaring operations

(a) $Sq^i\colon E_r^{s,t} \longrightarrow E_r^{s,t+i} \qquad (i < t),$

(b) $Sq^i\colon E_r^{s,t} \longrightarrow E_r^{s+i-t,2t} \qquad (i \geq t),$

for $p=2$, and the reduced power operations

(a) $\beta^\varepsilon P^i\colon E_r^{s,t} \longrightarrow E_r^{s,t+2i(p-1)+\varepsilon} \qquad (2i < t;\ \varepsilon = 0, 1),$

(b) $\beta^\varepsilon P^i\colon E_r^{s,t} \longrightarrow E_r^{s+(2i-t)(p-1)+\varepsilon,pt} \qquad (2i \geq t;\ \varepsilon = 0, 1),$

for p an odd prime; and they discussed the properties of these operations. Also L. Kristensen [6], [7] obtained the results by using the simplical method.

On the other hand, along with the establishment of the Eilenberg-Moore spectral sequence, J. P. May conjectured at the Conference on Algebraic Topology at Chicago Circle in 1968 that one might introduce the Steenrod operations into the mod p Eilenberg-Moore spectral sequence; and then D. Rector [10] and L. Smith [15], [16] showed that the mod p Eilenberg-Moore spectral sequence is a spectral sequence of modules over the mod p Steenrod algebra with respect to the operations of type (a).

Further, in his work [9], J. P. May developed a general theory to introduce the Steenrod operations into a spectral sequence, and W. M. Singer [14] introduced the squaring operations of both types (a) and (b) into a class of spectral sequences such as the change of ring spectral sequence, the Eilenberg-Moore spectral sequence and the Serre spectral sequence. It remains to introduce the Steenrod reduced powers into such spectral sequences.

The purpose of this paper is to introduce and study the Steenrod operations of both types (a) and (b) for any prime p in such a class of spectral sequences of Eilenberg-Moore type. The main results are Theorems 1.2, 1.3, 1.4, 1.5 and 1.6. Our results extend those obtained by W. M. Singer [14] who worked when $p=2$. The method is slightly different from [14]. The key lemma is Lemma