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Chain Conditions for Abelian, Nilpotent and Soluble Ideals in Lie Algebras

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1. Introduction

Let $\mathfrak{X}$ be a class of Lie algebras over a field $\mathfrak{k}$, and let Max-$\lhd\mathfrak{X}$ (resp. Min-$\lhd\mathfrak{X}$) be the class of Lie algebras which satisfy the maximal (resp. minimal) condition for $\mathfrak{X}$-ideals. Amayo and Stewart have asked the following among "Some open questions" in [1]: *Are there any inclusions between* Max-$\lhd\mathfrak{A}$, Max-$\lhd\mathfrak{N}$, Max-$\lhd$E$\mathfrak{A}$; Min-$\lhd\mathfrak{A}$, Min-$\lhd\mathfrak{N}$, Min-$\lhd$E$\mathfrak{A}$?

Recently it was shown by Kubo [2] that Max-$\lhd\mathfrak{A}$ and Max-$\lhd\mathfrak{N}$ (resp. Min-$\lhd\mathfrak{A}$ and Min-$\lhd\mathfrak{N}$) do not necessarily coincide with each other. He showed these facts by considering a certain Lie algebra over the rational number field.

The purpose of this paper is to show the following theorems.

THEOREM 1. *Over any field*

$$\text{Max-}\lhd\mathfrak{N} \gneqq \text{Max-}\lhd \text{E}\mathfrak{A} \quad \textit{and} \quad \text{Min-}\lhd\mathfrak{N} \gneqq \text{Min-}\lhd \text{E}\mathfrak{A}.$$

THEOREM 2. *Over any field*

$$\text{Max-}\lhd\mathfrak{A} \gneqq \text{Max-}\lhd\mathfrak{N}.$$

Throughout the paper, we shall employ the notations and terminology in [1].

2. Proof of Theorem 1

Let $\mathfrak{k}$ be an arbitrary field and A an infinite extension field of $\mathfrak{k}$. Let ρ be the regular representation of A. Consider A as an abelian Lie algebra over $\mathfrak{k}$, so that ρ becomes a Lie homomorphism of A into Der(A). Thus we can form the split extension

$$L = A \dotplus \rho(A),$$

where $A \lhd L$ and $[a, \rho(b)] = ab$ for any $a, b \in A$.

We first show that any non-zero ideal of L contains A. Suppose $0 \neq I \lhd L$. Then $0 \neq I \cap A \lhd L$. In fact, if $I \cap A = 0$, then there exist $a, b \in A$ with $b \neq 0$ such