HIROSHIMA MATH. J.
9 (1979), 335–346

Estimates for the Coincidence Sets of Solutions of Elliptic Variational Inequalities

Toshitaka NAGAI

(Received January 10, 1979)

1. Introduction

In this paper, we shall be concerned with the following elliptic variational inequalities with obstacle Ψ

$$\text{(VI)}\qquad \begin{cases} -\Delta u + \alpha u \geqq f & \text{in } \Omega, \\ u \geqq \Psi & \text{in } \Omega, \\ (u - \Psi)(-\Delta u + \alpha u - f) = 0 & \text{in } \Omega \end{cases}$$

under the three types of boundary conditions

$$\text{(DC)}\qquad u = \psi \quad \text{on } \Gamma,$$

$$\text{(NC)}\qquad \frac{\partial u}{\partial n} = \phi \quad \text{on } \Gamma$$

and

$$\text{(SC)}\qquad u \geqq \psi, \quad \frac{\partial u}{\partial n} \geqq \phi, \quad (u - \psi)\left(\frac{\partial u}{\partial n} - \phi\right) = 0 \quad \text{on } \Gamma,$$

where Ω is a bounded domain in R^N with smooth boundary Γ, n is the unit outer normal to Γ, Δ denotes the Laplace operator and α is a positive constant. The boundary conditions (DC), (NC) and (SC) are the Dirichlet condition, the Neumann condition and the Signorini condition, respectively. The variational inequalities (VI) have been investigated by many authors. For instance, we refer to the papers [3], [4] and [6]. Applications of the variational inequalities (VI) to physical problems have been given in [1] and [5].

Given a solution u of (VI), the domain Ω is divided into two parts Ω_1 and Ω_2 such that

$$\Omega_1 = \{x \in \Omega;\ u(x) = \Psi(x)\},$$

$$\Omega_2 = \{x \in \Omega;\ u(x) > \Psi(x)\}.$$

Ω_1 is called the coincidence set of u. It is of interest to give an estimate of the size of Ω_1. Recently, A. Bensoussan, H. Brézis and A. Friedman [2] gave an