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Subideality and Ascendancy in Generalized Solvable Lie Algebras

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Introduction

Wielandt [8] has given some criteria for a subgroup to be subnormal in a finite group. Peng [6, 7] and Hartley and Peng [3] have given similar criteria for not necessarily finite groups. Furthermore Chao and Stitzinger [2] have given conditions for a subalgebra to be a subideal in a finite-dimensional solvable Lie algebra.

In this paper we shall investigate some criteria for subideality and ascendancy in not necessarily finite-dimensional Lie algebras.

Let L be a Lie algebra over a field $\mathfrak{k}$ and let H be a subalgebra of L. When $L/\mathrm{Core}_L(H)$ is solvable, H is a subideal of L if either (a) there exists some integer $n \geq 0$ such that $[L, {}_nH] \subseteq H$, or (b) there exists some integer $n \geq 0$ such that $[L, {}_nx] \subseteq H$ for any $x \in H$ and the characteristic of $\mathfrak{k}$ is 0 or $p > n$ (Theorem 4 and Theorem 7). When $L/\mathrm{Core}_L(H)$ is hyperabelian, H is an ascendant subalgebra of L if one of the following conditions is satisfied: (c) For any $a \in L$ there exists an integer $n = n(a)$ such that $[a, {}_nH] \subseteq H$; (d) $\mathfrak{k}$ is of characteristic 0, H is solvable, and for any $a \in L$ there exists $n = n(a)$ such that $[a, {}_nx] \in H$ for any $x \in H$ (Theorem 12 and Theorem 14). Finally when $L/\mathrm{Core}_L(H)$ has an ascending abelian series, H is an ascendant subalgebra of L if $\langle a^H \rangle$ is finitely generated for any $a \in L$ and one of the above conditions (c) and (d) is satisfied (Theorem 17 and Theorem 18).

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1. Preliminaries

Throughout the paper Lie algebras are not necessarily finite-dimensional over a field $\mathfrak{k}$ of arbitrary characteristic unless otherwise specified. We mostly follow [1] for the use of notations and terminology.

Let L be a Lie algebra over $\mathfrak{k}$. L belongs to the class $\acute{\mathrm{E}}\mathfrak{A}$ if L has an ascending abelian series $(L_\alpha)_{\alpha \leq \lambda}$. If each L_α $(\alpha \leq \lambda)$ is furthermore an ideal of L, then L belongs to the class $\acute{\mathrm{E}}(\lhd)\mathfrak{A}$, that is, L is hyperabelian. For an integer $n \geq 0$ and an ordinal λ, $H \leq L$, $H \lhd L$, H si L, $H \lhd^n L$, H asc L and $H \lhd^\lambda L$ mean that H is respectively a subalgebra, an ideal, a subideal, an n-step subideal, an ascendant