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Remark on the dual of some Lipschitz spaces

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The present paper is considered as a supplement to the author's paper [1]. Our aim is to prove the following theorem.

THEOREM. *Let α be a real number, $1<p\leqq\infty$ and $1\leqq q\leqq\infty$.*

(a) *The dual of $\boldsymbol{\lambda}^{\alpha}_{\infty,q}$ is isomorphic to $\boldsymbol{\Lambda}^{-\alpha}_{1,q'}$.*

(b) *There exists an isomorphism η: $\boldsymbol{\Lambda}^{\alpha}_{p,\infty}\to(\boldsymbol{\lambda}^{\alpha}_{p,\infty})''$ such that the restriction of η to $\boldsymbol{\lambda}^{\alpha}_{p,\infty}$ is the canonical embedding of $\boldsymbol{\lambda}^{\alpha}_{p,\infty}$ into its second dual $(\boldsymbol{\lambda}^{\alpha}_{p,\infty})''$.*

Notation and related definitions are given in section 1. As in [1], the proof of the Theorem is done by establishing the corresponding results for some spaces of harmonic functions which are isomorphic to the spaces considered in the Theorem. Our result (a) is an n-dimensional and non-periodic version of a result of T. M. Flett [3; Theorem 19], whereas (b) when $p=\infty$ is that of a result of K. de Leeuw [5; Theorem 2.1] (cf. also [3; Theorem 19]).

1. Notation and preliminaries

We use R^n to denote the n-dimensional Euclidean space, and for each point $x=(x_1,\dots, x_n)$ we write $|x|=(x_1^2+\cdots+x_n^2)^{1/2}$.

Unless otherwise stated, all functions are supposed to be complex-valued. As usual we use $\mathscr{S}=\mathscr{S}(R^n)$ to denote the space of all rapidly decreasing functions on R^n; $\mathscr{D}$ stands for its subspace consisting of functions with compact supports.

For any positive integer k let Z_k^+ be the set of all ordered k-tuples of non-negative integers, and for each $\mu=(\mu_1,\dots, \mu_k)$ let

$$|\mu| = \mu_1 + \cdots + \mu_k.$$

An element of Z_k^+ is called a multi-index.

If u is a function defined on an open subset of R^k, we use D_i^m to denote the partial derivative of u of order m with respect to the i-th coordinate. Further, for each multi-index $\mu=(\mu_1,\dots, \mu_k)$ we write

$$D^{\mu}u = D_1^{\mu_1}\cdots D_k^{\mu_k}u.$$

If f is a measurable function defined on R^n, we set