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# Openness of loci, P-excellent rings and modules

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## Introduction

The study of the problem of openness of certain loci over noetherian rings was firstly undertaken systematically by A. Grothendieck [7]. There he has initiated the theory of *excellent rings*, where the important notion is that of the fibers of homomorphisms (e.g., formal fibers). In recent years, R. Y. Sharp has developed the theory of *acceptable rings* replacing regularity in the definition of excellent rings by Gorensteinness (cf. [15], [16]). On the other hand, some authors recently have recognized the importance of the so-called *Nagata's criterion for openness of loci* and they have proved it for many important properties (cf. [6], [8]).

The aim of this paper is to generalize the above-mentioned theories for rings to modules and to study the problem of openness of certain loci of finitely generated modules over noetherian rings. In particular, we shall present a theory of P-*excellent rings* and P-*excellent modules* (where P is a given property for finitely generated modules over noetherian local rings) which includes the theories of excellent rings and acceptable rings as special cases. For this purpose, we have to extend the definitions and axioms which have been considered only for rings by the above authors to modules and we shall generalize their results to modules.

The contents are divided into two parts. In Part I, we shall study the problem of openness of loci and the theory of P-*excellent modules* axiomatically. Concerning general properties on P-*morphisms* and *well-fibered modules* (cf. Def. 6, Def. 12), we shall mainly follow the method due to R. Y. Sharp [15]. Firstly we shall prove a fundamental theorem on P-excellent modules: P-excellent modules are stable under homomorphisms essentially of finite type. Next, we shall show the following theorems under certain conditions on P (which are satisfied for P = Cohen-Macaulay, Gorenstein and complete intersection):

(1) All modules over a P-excellent ring are P-excellent.

(2) If an $A$-module $M$ is P, then $M$ is P-excellent.

(3) Suppose that there is an $A$-module $M$ which is P and that Supp$(M)$ coincides with Spec$(A)$. Then $A$ is P-excellent.

Lastly, we shall treat the finite descent of P-excellent rings which generalizes a theorem of S. Greco [5] on excellent rings.