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Global solvability of the Laplace operator on a non-compact affine symmetric space

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1. Introduction

A. Cerezo and F. Rouvière prove that the Casimir operator on a complex semisimple Lie group G is surjective on $C^\infty(G)$ ([1]). Further J. Rauch and D. Wigner prove the global solvability of the Casimir operator when G is a non-compact semisimple Lie group with finite center ([6]). S. Helgason proves that each invariant differential operator on a symmetric space X of the non-compact type is surjective on $C^\infty(X)$ ([3]).

In this paper, we will show that the Laplace operator on an affine symmetric space induced by the Casimir operator is globally solvable by means of the method given by J. Rauch and D. Wigner ([6]).

2. Notation and preliminaries

Let M be an infinitely differentiable manifold. We denote by $C^\infty(M)$, $C_c^\infty(M)$, $\mathfrak{X}(M)$, and $\Omega^1(M)$ the space of infinitely differentiable functions on M, the space of infinitely differentiable functions on M with compact support, the space of all smooth vector fields on M and the space of all smooth 1-forms on M, respectively.

Let G be a non-compact connected real semisimple Lie group with finite center, $\mathfrak{g}$ the Lie algebra of G, and B the Killing form of $\mathfrak{g}$. Let σ be an involution of G, G_σ the closed subgroup of G consisting of all the elements left fixed by σ, and H a closed subgroup of G lying between G_σ and the identity component of G_σ. Then the homogeneous space G/H is said to be an affine symmetric space and there always exists a G-invariant measure dgH on G/H which is unique except for a strictly positive factor of proportionality. The eigenvalues of the involution of $\mathfrak{g}$ induced by σ are 1 and -1. Let $\mathfrak{h}$ be the eigenspace for 1 and $\mathfrak{q}$ the eigenspace for -1. The direct decomposition of $\mathfrak{g}$ is: $\mathfrak{g}=\mathfrak{h}+\mathfrak{q}$. Since there exists a Cartan involution of $\mathfrak{g}$ commuting with σ, $\mathfrak{g}$ decomposes into a vector space direct sum:

$$\mathfrak{g} = \mathfrak{q} \cap \mathfrak{k} + \mathfrak{h} \cap \mathfrak{k} + \mathfrak{q} \cap \mathfrak{p} + \mathfrak{h} \cap \mathfrak{p}$$

where $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ is the Cartan decomposition. Thus we can choose a basis X_1,