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Asymptotic analysis of odd order ordinary differential equations

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1. Introduction

In this paper we consider the differential equations

$$L_n x + q(t)x = 0, \tag{1}$$

$$L_n x + q(t)f(t, x) = 0, \tag{2}$$

where $n \geq 3$ is an odd number and L_n is the differential operator of the form

$$L_n = \frac{1}{p_n(t)} \frac{d}{dt} \frac{1}{p_{n-1}(t)} \frac{d}{dt} \cdots \frac{d}{dt} \frac{1}{p_1(t)} \frac{d}{dt} \frac{\cdot}{p_0(t)}. \tag{3}$$

The following conditions are always assumed to hold:

(i) $p_i(t)$ $(0 \leq i \leq n)$ and $q(t)$ are continuous and positive on the interval $[a, \infty)$, and

$$\int_a^\infty p_i(t)dt = \infty \qquad \text{for} \quad 1 \leq i \leq n-1.$$

(ii) $f(t, x)$ is continuous on $[a, \infty) \times R$, $f(t, x)$ is nondecreasing in x and $xf(t, x) > 0$ for $x \neq 0$.

We introduce the notation:

$$\begin{aligned} &D^0(x; p_0)(t) = \frac{x(t)}{p_0(t)}, \\ &D^j(x; p_0,\dots, p_j)(t) = \frac{1}{p_j(t)} \frac{d}{dt} D^{j-1}(x; p_0,\dots, p_{j-1})(t), \quad 1 \leq j \leq n. \end{aligned} \tag{4}$$

Then the differential operator L_n can be rewritten as

$$L_n = D^n(\cdot\,; p_0,\dots, p_n).$$

The domain $\mathscr{D}(L_n)$ of L_n is defined to be the set of all functions $x: [T_x, \infty) \to R$ such that $D^j(x; p_0,\dots, p_j)(t)$ $(0 \leq j \leq n)$ exist and are continuous on $[T_x, \infty)$.

A nontrivial solution of (1) (or (2)) is called oscillatory if the set of its zeros is infinite. Otherwise, it is called nonoscillatory. A nontrivial solution $x(t)$ of (1) (or (2)) is said to be strongly decreasing if it satisfies