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Invariant sequences in Brown-Peterson homology and some applications

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§0. Introduction

Let BP be the Brown-Peterson ring spectrum at p, where p is a prime number. Then

$$BP_* = Z_{(p)}[v_1, v_2, \dots], \quad \dim v_n = 2(p^n - 1),$$

where the v_n's are Hazewinkel's generators. A sequence of elements $a_0, a_1, \dots, a_s$ of BP_* is said to be *invariant* if

$$\eta_R a_i = \eta_L a_i \bmod (a_0, a_1, \dots, a_{i-1}) \cdot BP_*BP \qquad \text{for} \quad i = 0, 1, \dots, s,$$

where $\eta_R, \eta_L \colon BP_* \to BP_*BP$ are the right and the left units of the Hopf algebroid BP_*BP over BP_*.

The purpose of this note is to prove the following

THEOREM 1.5. *Let $s_0, s_1, \dots, s_n$ be positive integers, and let p^{e_i} be the largest power of p dividing s_i. Then the sequence $p^{s_0}, v_1^{s_1}, \dots, v_n^{s_n}$ is invariant if and only if $s_0 - 1 \leqq e_1$ and $s_i \leqq p^{e_{i+1} - s_0 + 1}$ for $i = 1, \dots, n-1$.*

The case $s_0 = 1$ of this theorem has been given by Baird [4; Lemma 7.6].

As an application, we obtain some γ-elements in $H^3 BP_*$ of order p^{s_0} in Corollary 2.5 (p: odd prime). Furthermore, we consider the non-realizability of some cyclic BP_*-modules in Corollary 2.7.

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§1. Invariant sequences in BP_*

Let p be a prime number, and let BP denote the Brown-Peterson ring spectrum at p. Then, it is known that

$$BP_* = Z_{(p)}[v_1, v_2, \dots, v_n, \dots], \quad \dim v_n = 2(p^n - 1),$$

where the v_n's are Hazewinkel's generators, and the Hopf algebroid

$$BP_*BP = BP_*[t_1, t_2, \dots, t_n, \dots], \quad \dim t_n = 2(p^n - 1),$$