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The order of the canonical elememt in the J-group of the lens space

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§1. Statement of the result

The standard lens space mod m is the orbit manifold

$$L^n(m) = S^{2n+1}/Z_m \qquad (Z_m = \{z \in S^1 : z^m = 1\})$$

of the $(2n+1)$-sphere $S^{2n+1}(\subset C^{n+1})$ by the diagonal action $z(z_0,\dots, z_n)=(zz_0, \dots, zz_n)$. Let η be the *canonical complex line bundle over* $L^n(m)$, i.e., the induced bundle of the canonical complex line bundle over the complex projective space $CP^n = S^{2n+1}/S^1$ by the natural projection $L^n(m) \to CP^n$.

Then, the purpose of this note is to prove the following

THEOREM 1.1. *Let p be an odd prime and r a positive integer. Then, the order of the J-image*

$$J(r\eta - 2) \in \tilde{J}(L^n(p^r))$$

of the stable class of the real restriction $r\eta$ of the canonical line bundle η is equal to

$$p^{f(n,r)}, \quad f(n,r) = \max\{s + [n/p^s(p-1)]p^s : 0 \leqq s < r \text{ and } p^s(p-1) \leqq n\},$$

where $f(n,r) = \max \emptyset = 0$ if $n < p-1$.

We notice that the above theorem is valid also for the case $p=2$ and $r \geqq 2$, by the result in the forthcoming paper [2].

It is proved by J. F. Adams [1] and D. Quillen [4] that

$$J(X) \cong KO(X)/\textstyle\sum_k (\cap_e k^e(\Psi^k - 1)KO(X))$$

(X: finite dimensional CW-complex) where Ψ^k is the Adams operation. Based on this result, we prove the theorem in §2 and study more generally the order of $Jr(\eta^i - 1)$ $(i \geqq 1)$ in §3, by using the partial results obtained in [3].

§2. Proof of Theorem 1.1

Let p be an odd prime. Consider the $2n$-skeleton