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Existence of non-tangential limits of solutions of non-linear Laplace equation

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Our aim in this note is to study the boundary behavior of (weak) solutions of the non-linear Laplace equation

$$(1) \qquad -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(|\mathrm{grad}\, u|^{p-2} \frac{\partial u}{\partial x_i}\right) = 0 \qquad \text{on} \quad \Omega,$$

where Ω is a domain in the n-dimensional Euclidean space R^n.

We say that $\xi \in \partial\Omega$ satisfies the interior cone condition if there is an open truncated cone Γ in Ω with vertex at ξ. Let F be the set of all $\xi \in \partial\Omega$ satisfying the interior cone condition. We can show that F is an F_σ-set*).

A function u on Ω is said to have a non-tangential limit at $\xi \in F$ if for any open truncated cone $\Gamma \subset \Omega$ with vertex at ξ,

$$\lim_{x \to \xi, x \in \Gamma'} u(x)$$

exists and is finite whenever Γ' is a cone with vertex at ξ whose closure $\bar{\Gamma}'$ is included in $\Gamma \cup \{\xi\}$.

In this note let $1 < p < \infty$ and let $\rho(x)$ denote the distance of x from $R^n - \Omega$.

THEOREM. *Let* $1 < p \leqq n$ *and let* u *be a function satisfying the following properties*:

i) *u is continuous on Ω*;

ii) *u is p-precise**) on any relatively compact open subset of Ω*;

iii) *u satisfies* (1) *in the weak sense* (*cf.* [4]);

iv) $\int_\Omega |\mathrm{grad}\, u(x)|^p \rho(x)^\alpha dx < \infty$ *for* $\alpha < p$.

Then there exists a set $E \subset \partial\Omega$ such that $B_{1-\alpha/p,p}(E) = 0$ and u has a non-tangential limit at each point of $F - E$.

Here $B_{1-\alpha/p,p}$ denotes the Bessel capacity of index $(1-\alpha/p, p)$ (see [1]). In case $p = 2$, our theorem is shown in [3; Theorem 2′].

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*) This fact was pointed out by Professor Makoto Sakai.

**) For the definition of p-precise functions, see Ziemer [5].