HIROSHIMA MATH. J.
10 (1980), 351–363

On glueings of prime ideals

Dedicated to Professor Yoshikazu Nakai on his sixtieth birthday

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(Received December 13, 1979)

In the article [11] Traverso defined the notion of seminormal subrings of a commutative ring and developed an interesting theory on such subrings. In particular, to study the structure of seminormal subrings of a noetherian ring, he used a nice method by which a subring A' is constructed from a ring B by glueing prime ideals $\mathfrak{p}_1,\ldots, \mathfrak{p}_n$ of B lying over a prime ideal $\mathfrak{p}$ of a subring A of B (for the precise definition see § 1). Such a subring A' is called the ring obtained from B by glueing over $\mathfrak{p}$, or simply a glueing of prime ideals $\mathfrak{p}_1,\ldots, \mathfrak{p}_n$. Traverso showed that any seminormal subring A in a noetherian ring B is obtained from B by a finite number of glueings, if B is a finite A-module (cf. Theorem 2.1 in [11]).

The aim of this paper is to show some results on glueings of prime ideals in the above sense. In § 1 we give a necessary and sufficient condition for a finite number of prime ideals $\mathfrak{p}_1,\ldots, \mathfrak{p}_n$ of a noetherian ring B to be glued. In other words, we give a condition under which there is a subring A of B such that B is a finite A-module and that $\mathfrak{p}_1,\ldots, \mathfrak{p}_n$ are the prime ideals of B lying over a prime ideal $\mathfrak{p}$ of A. Moreover we investigate when the ring A' obtained from B by glueing over $\mathfrak{p}$ coincides with A. Although Pedrini [8] and Tamone [10] have already attacked the same problem, the rings treated by them are very special ones. So, in § 2, we apply our results in § 1 to these special cases and show how our results work there in a unified way. Next, in § 3, we show that Serre's property (S_2) goes down from a noetherian ring B to a glueing A of prime ideals of height 1. This result also has been shown by Pedrini [8] in very special cases of integral domains, but we give a complete proof of this without any assumption. Moreover we show that if A is a glueing of prime ideals $\mathfrak{p}_1,\ldots, \mathfrak{p}_n$ of a noetherian ring B one of which has height >1 and if any $\mathfrak{p}_i$ contains a regular element of B, then A does not have the property (S_2) or A coincides with B. In the last section we study local rings which are glueings of maximal ideals of semilocal rings. When A is a local subring of a semilocal ring B which is finite over A, we give several conditions for A to be a glueing of maximal ideals of B. In particular a condition for A to be such a glueing of a regular semilocal ring B will be given in terms of multiplicity of A and the conductor of A in B in the case where A is the locality of a closed point of an algebraic variety.

All the rings in this paper are commutative with unit.