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Estimates on the support of solutions of parabolic variational inequalities in bounded cylindrical domains

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1. Introduction

Consider a solution u of the following Cauchy problem of the parabolic variational inequality:

$$\frac{\partial u}{\partial t} - \Delta u \leqq f, \qquad u \leqq 0,$$

$$u\left(\frac{\partial u}{\partial t} - \Delta u - f\right) = 0 \qquad \text{in} \quad R^n \times]0, T[,$$

$$u(x, 0) = u_0(x) \qquad \text{in} \quad R^n.$$

The support $S(t)$ of the function $x \to u(x, t)$ has been studied by Bensoussan and Lions [2], Brezis and Friedman [4] and Evans and Knerr [6]. They proved that

$$S(t) \subset S(0) + B(c(t|\log t|)^{1/2})$$

for sufficiently small $t>0$, where $+$ denotes the vector sum, $B(\rho)=\{x \mid |x| \leqq \rho\}$, and c is a positive constant.

Some results which conclude

$$u(x, t) = 0 \qquad \text{for} \quad t > (\text{some constant}), \ x \in R^n,$$

or

$$u(x, t) = 0 \qquad \text{for} \quad |x| > (\text{some constant}), \ t > 0$$

are stated in the book by Bensoussan and Lions [3, Chapter 3, § 2.16].

In this paper we shall consider a solution u of the parabolic variational inequality with Dirichlet boundary condition

$$\frac{\partial u}{\partial t} - \Delta u \leqq f, \qquad u \leqq 0,$$

$$u\left(\frac{\partial u}{\partial t} - \Delta u - f\right) = 0 \qquad \text{in} \quad \Omega \times]0, T[,$$