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Euler integral transformations of hypergeometric functions of two variables

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§0. Introduction

It is known that the hypergeometric functions F_1, F_2, F_3 and F_4 of two variables have the following Euler integral representations

$$(0.0)\quad F_1(\alpha,\beta,\beta',\gamma;x,y) = C_0\int_{D_0} u^{\alpha-1}(1-u)^{\gamma-\alpha-1}(1-ux)^{-\beta}(1-uy)^{-\beta'}du,$$

$$(0.1)\quad F_1(\alpha,\beta,\beta',\gamma;x,y) = C_1\iint_{D_1} u^{\beta-1}v^{\beta'-1}(1-u-v)^{\gamma-\beta-\beta'-1}(1-ux-vy)^{-\alpha}dudv,$$

$$(0.2)\quad F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y)$$
$$= C_2\iint_{D_2} u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}dudv,$$

$$(0.3)\quad F_3(\alpha,\alpha',\beta,\beta',\gamma;x,y)$$
$$= C_3\iint_{D_3} u^{\beta-1}v^{\beta'-1}(1-u-v)^{\gamma-\beta-\beta'-1}(1-ux)^{-\alpha}(1-vy)^{-\alpha'}dudv,$$

$$(0.4)\quad F_4(\alpha,\beta,\gamma,\gamma';x,y) = C_4\iint_{D_4} u^{\alpha-\gamma'}v^{\alpha-\gamma}(u+v-uv)^{\gamma+\gamma'-\alpha-2}(1-ux-vy)^{-\beta}dudv,$$

where C_j are some constants and D_j are some cycles. (0.0) is discovered by E. Picard, (0.4) is discovered by K. Aomoto (see [3]) and the others are discovered by P. Appell.

In this paper, we establish a principle of Euler integral representations and give new integral formulae for the hypergeometric functions F_2, G_1, G_2, H_1 and H_2 (Horn's notation).

THEOREM.

$$(0.6)\quad F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y)$$
$$= C_6\iint_{D_6} u^{\alpha-\gamma'}v^{\alpha-\gamma}(u+v-uv)^{\gamma+\gamma'-\alpha-2}(1-ux)^{-\beta}(1-vy)^{-\beta'}dudv,$$

$$(0.7)\quad G_1(\alpha,\beta,\beta';x,y) = C_7\int_{D_7} u^{\beta'-1}(u+1)^{-\beta-\beta'}(1-ux-\frac{y}{u})^{-\alpha}du,$$