

Multiplicity one fails for p -adic unitary principal series

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For a real semisimple group of matrices, every unitary principal series representation splits into inequivalent irreducible representations [5]. This multiplicity-one result was conjectured [7] in 1971 because one expected in general and knew in some special cases that all reducibility was accounted for by canonical geometric constructions, such as the imbedding of a space of analytic functions on the disc in the space of functions on the circle by passage to boundary values.

We shall give an example to show that the corresponding multiplicity-one statement is false for a semisimple group of matrices defined over a locally compact, totally disconnected, nondiscrete field of characteristic 0. This example is summarized in §2, and its properties are verified in §5. It is ultimately motivated by the work of Langlands [11] on classification of irreducible admissible representations. Langlands [12] was able to give a formulation of the results of [8] that suggests that straightforward generalization of the multiplicity-one theorem to other fields is not likely to succeed. An exposition of [12] is given in [9], and the way in which this work motivates our example is explained in §6.

Verification of the properties of our example depends on a suitable development of intertwining operators for split groups. Most of such a development has been carried out by Sally [15] and Winarsky [18]. Some small modifications and elaborations of their work are the subject of §3 and 4.

The work in this paper grew out of conversations at the American Mathematical Society Summer Institute in 1977 with Langlands, Lusztig, Schiffmann, Shelstad, and Wallach. The work by I. Muller [13] on intertwining operators was also of influence; Muller developed a variation on Winarsky's work and was able to push through analogs of the results of [8]. In addition, she came close to discovering the example of §2. We thank all these people for their help.

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