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On the oscillation of solutions of forced even order nonlinear differential equations

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The oscillation of even order differential equations, both forced and unforced, has been an area of a large amount of interest. For example, see [4] and its bibliography.

In this paper we will give some results concerning the oscillation of solutions to equations of the form

$$x^{(2n)} + f(t, x, x', \dots, x^{(2n-1)}) = R^{(2n)}(t) \tag{1}$$

where the functions f and R satisfy appropriate conditions. Our conditions on R generalize those found in [2] and [3].

A solution to (1) on an interval $[a, \infty)$ is said to be oscillatory if it has an unbounded set of zeros. A real valued function R is called *strongly bounded* if it assumes its maximum and minimum on every interval of the form $[a, \infty)$, $0<a$. Throughout the remainder of this paper $\mathbf{R}$ and $\mathbf{R}^+$ will denote the reals and nonnegative reals respectively.

LEMMA 1. *Let* $R \in C^{2n}[\mathbf{R}^+, \mathbf{R}]$ *be strongly bounded and* $f \in C^0[\mathbf{R}^+ \times \mathbf{R}^{2n}, \mathbf{R}]$ *be such that* $x_1 f(t, x_1, \dots, x_{2n}) \geq 0$ *for every* $t \geq 0$ *and* $(x_1, \dots, x_{2n}) \in \mathbf{R}^{2n}$. *If* x *is a bounded solution of* $x^{(2n)} + f(t, x, x', \dots, x^{(2n-1)}) = R^{(2n)}(t)$ *on an interval* $[a, \infty)$, *then exactly one of the following holds*:

(i) *x is oscillatory,*

(ii) *there is a* $b>0$ *such that* $0<x(t)$ *and*

$$(-1)^k[x^{(k)}(t) - R^{(k)}(t)] \leq 0 \text{ for } k = 1, 2, \dots, 2n \text{ on } [b, \infty),$$

(iii) *there is a* $b>0$ *such that* $x(t)<0$ *and*

$$(-1)^k[x^{(k)}(t) - R^{(k)}(t)] \geq 0 \text{ for } k = 1, 2, \dots, 2n \text{ on } [b, \infty).$$

If x is any nonoscillatory solution on an interval $[a, \infty)$, *then*

(iv) *there are* c, $C>0$ *such that* $C<|x(t)|$ *whenever* $c \leq t$.

Moreover, if x is an unbounded nonoscillatory solution, then $|x(t)| \to \infty$ *as* $t \to \infty$.