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A subspace of Schwartz space on motion groups

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§1. Introduction

In the theory of harmonic analysis on semisimple Lie groups, it is important to consider the space $\mathscr{C}^p$, $0<p\leqq 2$, which is an L^p type subspace of the Schwartz space $\mathscr{C}=\mathscr{C}^2$, and one of the most important problems at present is to determine the image of $\mathscr{C}^p$ by the Fourier transform. For example, if we consider the space $\mathscr{C}^p(X)$ on a symmetric space X, then the image of $\mathscr{C}^p(X)$ is the space of holomorphic functions in the interior of a certain tube domain of a complex space satisfying some boundedness conditions modulo representations of a compact group (see M. Eguchi [1], Theorem 4.8.1). In the present paper we consider the corresponding space to $\mathscr{C}^p$ for the motion groups.

Let K be a compact connected Lie group acting on a finite dimensional real vector space V as a linear group. Let G be the semidirect product group of V and K. We call this group the motion group. Let $\hat{V}$ be the dual space of V and $\hat{V}_c$ the complexification of $\hat{V}$. We fix a K-invariant inner product $(\ ,\ )$ of V, an orthonormal basis of V with respect to this inner product and its dual basis. We identify V and $\hat{V}$ with $\boldsymbol{R}^n$ by these bases. Let $x=(x_1,\dots, x_n)\in V$ and $\xi=(\xi_1,\dots, \xi_n)\in\hat{V}$, where $n=\dim V$. We put $|x|^2=(x, x)$. Then $|x|^2=x_1^2+\cdots+x_n^2$. We also put $|\xi|^2=\xi_1^2+\cdots+\xi_n^2$. For any $\varepsilon>0$ we define the tube domain F^ε by setting

$$F^\varepsilon = \{\zeta = \xi+i\eta\in\hat{V}+i\hat{V}=\hat{V}_c\,;\,|\eta|\leqq\varepsilon\},$$

where $i=(-1)^{1/2}$. We denote by $\operatorname{Int}F^\varepsilon$ the interior of F^ε. We put $F^0=\operatorname{Int}F^0=\hat{V}$. Then F^ε and $\operatorname{Int}F^\varepsilon$ are K-invariant. Let $\mathfrak{H}=L^2(K)$ be the Hilbert space of square integrable functions on K with respect to the normalized Haar measure dk. Let $\boldsymbol{B}(\mathfrak{H})$ be the Banach space of all bounded linear operators on $\mathfrak{H}$. For $\varepsilon>0$ we denote by $\mathscr{Z}(F^\varepsilon)$ the set of all $\boldsymbol{B}(\mathfrak{H})$-valued C^∞ functions T on $\hat{V}$ which satisfy the following conditions:

(i) The function T extends holomorphically to $\operatorname{Int}F^\varepsilon$;

(ii) for any $\alpha\in\boldsymbol{N}^n$, $\ell\in\boldsymbol{N}$ and for any right invariant differential operators y, y' on K

$$\sup_{\zeta\in\operatorname{Int}F^\varepsilon}(1+|\zeta|^2)^\ell\|yD_\zeta^\alpha T(\zeta)y'\| < \infty, \tag{1.1}$$

where $D_\zeta^\alpha=\partial^{|\alpha|}/\partial\zeta_1^{\alpha_1}\cdots\partial\zeta_n^{\alpha_n}$ $(\alpha=(\alpha_1,\dots,\alpha_n),\ |\alpha|=\alpha_1+\cdots+\alpha_n)$;

(iii) for all $k\in K$ and for all $\zeta\in\operatorname{Int}F^\varepsilon$