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J-groups of lens spaces modulo powers of two

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§1. Introduction

Let $J(X)$ be the J-group of a CW-complex X of finite dimension. Then by J. F. Adams [2] and D. Quillen [10], it is shown that

$$(1.1) \qquad J(X) = KO(X)/\mathrm{Ker}\, J, \quad \mathrm{Ker}\, J = \textstyle\sum_k (\bigcap_e k^e(\Psi^k - 1)KO(X)),$$

where $KO(X)$ is the KO-group of X, $J\colon KO(X) \to J(X)$ is the natural epimorphism and Ψ^k is the Adams operation.

In this paper, we study the J-group of the standard lens space modulo 2^r $(r \geqq 2)$:

$$L^n(2^r) = S^{2n+1}/Z_{2^r}, \quad Z_{2^r} = \{z \in S^1\colon z^{2^r} = 1\},$$

which is the orbit manifold of the unit $(2n+1)$-sphere S^{2n+1} in C^{n+1} by the diagonal action $z(z_0,\dots, z_n) = (zz_0,\dots, zz_n)$. In the case $r=1$, $L^n(2)$ is the real projective space RP^{2n+1}, and its J-group $J(L^n(2))$ is determined by J. F. Adams ([1, Th. 7.4], [2, II, Ex. (6.3)]).

Let η be the canonical complex line bundle over $L^n(2^r)$, i.e., the induced bundle of the canonical complex line bundle over the complex projective space $CP^n = S^{2n+1}/S^1$ by the natural projection $L^n(2^r) \to CP^n$. Then, the main purpose of this paper is to prove the following

THEOREM 1.2. *Let $r \geqq 2$ and let $r(\eta^i - 1) \in \widetilde{KO}(L^n(2^r))$ be the real restriction of the stable class of the i-fold tensor product $\eta^i = \eta \otimes \cdots \otimes \eta$ of the canonical complex line bundle η over $L^n(2^r)$. Then the order of the J-image*

$$Jr(\eta^i - 1) \in \tilde{J}(L^n(2^r))$$

is equal to

$$2^{f(n,r;v)}, \quad f(n,r;v) = \max\{s - v + [n/2^s]2^{s-v}\colon v \leqq s < r \text{ and } 2^s \leqq n\},$$

where $v = v_2(i)$ is the exponent of 2 in the prime power decomposition of i and $\max \emptyset = 0$.

Recently, we have proved in [5, Th. 1.1, 3.1] that the above theorem is valid