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On the behavior at infinity of Green potentials in a half space

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1. Introduction

Let u be a Green potential in the half space $D=\{x=(x_1,\dots, x_n);\ x_n>0\}$, $n\geqq 2$. Then it is known that $x_n^{-1}u(x)$ tends to zero as $|x|\to\infty$, $x\in D-E$, where E is minimally thin at infinity (cf. [2]). Recently Essén-Jackson [2] have proved that $|x|^{-1}u(x)$ tends to zero as $|x|\to\infty$, $x\in D-E$, where the exceptional set E is called rarefied at infinity. Our aim in this note is to extend these results to Green potentials of general order.

Let k be a non-negative Borel measurable function on $R^n\times R^n$, and set

$$k(x, \mu) = \int_E k(x, y)d\mu(y) \quad \text{and} \quad k(\mu, y) = \int_E k(x, y)d\mu(x)$$

for a non-negative measure μ on a Borel set $E\subset R^n$. We define a capacity C_k by

$$C_k(E) = \sup \mu(R^n), \qquad E\subset D,$$

where the supremum is taken over all non-negative measures μ such that S_μ (the support of μ) is contained in E and

$$k(x, \mu) \leqq 1 \qquad \text{for every} \quad x\in D.$$

Let G_α be the Green function of order α for D, i.e.,

$$G_\alpha(x, y) = \begin{cases} |x-y|^{\alpha-n} - |\bar{x}-y|^{\alpha-n} & \text{in case} \quad 0<\alpha<n, \\ \log(|\bar{x}-y|/|x-y|) & \text{in case} \quad \alpha=n, \end{cases}$$

where $\bar{x}=(x_1,\dots, -x_n)$ for $x=(x_1,\dots, x_n)$. For $0\leqq\beta\leqq 1$, we consider the function $k_{\alpha,\beta}$ defined by

$$k_{\alpha,\beta}(x, y) = \begin{cases} x_n^{-1}y_n^{-\beta}G_\alpha(x, y) & \text{for } x, y\in D, \\ \lim_{z\to x, z\in D} z_n^{-1}y_n^{-\beta}G_\alpha(z, y) = a_\alpha y_n^{1-\beta}|x-y|^{\alpha-n-2} & \\ & \text{for } x\in\partial D \text{ and } y\in D, \end{cases}$$

where $a_\alpha=2(n-\alpha)$ if $\alpha<n$ and $=2$ if $\alpha=n$. In case $\beta=1$, $k_{\alpha,1}$ is extended to be continuous on $\bar{D}\times\bar{D}$ in the extended sense.