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# Subnormality and ascendancy in groups

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## Introduction

Recently subnormality in groups was investigated by Wielandt [7], Peng [4, 5], McCaughan and McDougall [3], and subnormality and ascendancy in groups were investigated by Hartley and Peng [1]. On the other hand, subideality and ascendancy in Lie algebras were examined by Kawamoto [2] and the author [6].

In this paper, following the paper [6] we shall introduce two notions of weak subnormality and weak ascendancy for subgroups, study their properties, and investigate several criteria for subnormality and ascendancy of subgroups.

Let $H$ be a subgroup of a group $G$. We shall show that when either (a) $G$ is hyperabelian, (b) $G$ has an ascending abelian series and $H$ is finite, or (c) $G$ is finite-by-hyperabelian and $H$ is finite, $H$ is ascendant in $G$ if and only if $H$ is weakly ascendant in $G$ (Theorems 3 and 7). Similar results for subnormality will be shown in Theorems 3 and 6. We shall also give characterizations of weak subnormality and $\omega$-step weak ascendancy (Theorem 4), and show that every finite, weakly ascendant subgroup of a group is at most of $\omega$-step (Theorem 5).

## 1.

Let $G$ be a group. If $x$, $y$ are elements of $G$, then $[x, y]=x^{-1}y^{-1}xy$ and we write $[x, {}_0y]=x$, $[x, {}_{n+1}y]=[[x, {}_ny], y]$ for an integer $n\geq 0$. If $X$, $Y$ are non-empty subsets of $G$, $[X, Y]$ is the set of all $[x, y]$ with $x\in X$ and $y\in Y$ and we write $[X, {}_0Y]=X$, $[X, {}_{n+1}Y]=[[X, {}_nY], Y]$.

We write $H\leq G$ if $H$ is a subgroup of $G$ and $H\lhd G$ if $H$ is a normal subgroup of $G$. For any ordinal $\lambda$, a subgroup $H$ of $G$ is a $\lambda$-step ascendant subgroup of $G$, denoted by $H\lhd^{\lambda}G$, if there is a series $(S_\alpha)_{\alpha\leq\lambda}$ of subgroups of $G$ such that

(a) $S_0=H$ and $S_\lambda=G$,

(b) $S_\alpha\lhd S_{\alpha+1}$ for any ordinal $\alpha<\lambda$,

(c) $S_\beta=\bigcup_{\alpha<\beta}S_\alpha$ for any limit ordinal $\beta\leq\lambda$.

$H$ is an ascendant subgroup of $G$ if $H\lhd^{\lambda}G$ for some ordinal $\lambda$. When $\lambda<\omega$, $H$ is a subnormal subgroup of $G$, denoted by $H$ sn $G$.

We say a subgroup $H$ of $G$ to be a $\lambda$-step weakly ascendant subgroup of $G$, if there is an ascending series $(S_\alpha)_{\alpha\leq\lambda}$ of subsets of $G$ satisfying the above conditions (a), (c) and the following condition: