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Wu classes and unoriented bordism classes of certain manifolds

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§1. Introduction

Let M be a closed manifold, and let w_i and v_i be the ith Stiefel-Whitney class and the ith Wu class of M, respectively. Then, the Wu formula means that they are related by the equality

$$v_n = \sum_{i=1}^{n} \theta^{n-i} w_i \tag{1.1}$$

(cf. Proposition 3.2), *where* $\theta^l = c(Sq^l) \in \mathscr{A}(2)$ *is the conjugation of* Sq^l *given in* [7, II, §4] *and is defined inductively by*

$$\theta^l = Sq^l + \sum_{i=1}^{l-1} Sq^i \theta^{l-i} = Sq^l + \sum_{j=1}^{l-1} \theta^{l-j} Sq^j \qquad (l \geqq 0).$$

The main purpose of this paper is to study the Wu classes by using (1.1).

To do this, we study the element θ^l in §2, and prove the following basic formula (Theorem 2.4), where we use always the notation

$$t' = 2^{t-1} \quad \text{for any positive integer } t:$$

(1.2) *If* $n = 2^k - 1$, *then*

$$\theta^n = Sq^{k'} Sq^{(k-1)'} \cdots Sq^1;$$

and if $n = 2^k - 1 - t_1' - \cdots - t_l'$ *with* $k \geqq t_1 > \cdots > t_l \geqq 1$, *then*

$$\theta^n = \sum_{1 \leqq p_1 < \cdots < p_l \leqq k} Sq^{I(p_1, \dots, p_l)},$$

where $I(p_1, \dots, p_l) = (i_1, \dots, i_k)$ *is given by*

$$i_{p_s} = (k - p_s + 1)' - t_s' \quad (s = 1, \dots, l), \qquad i_p = (k - p + 1)' \quad (p \neq p_1, \dots, p_l),$$

and $Sq^{(i_1, \dots, i_k)} = Sq^{i_1} \cdots Sq^{i_k}$ *with* $Sq^0 = 1$ *and* $Sq^i = 0$ *for* $i < 0$.

As an application of this formula, we see the well known formula

$$\theta^{2n+1} = \theta^{2n} Sq^1$$

(Corollary 2.14) and the one given by D. M. Davis [2, Th. 2] (Corollary 2.16). By using the former, we can reduce the equality (1.1) to the form given in Theorem 3.9, and we obtain the equality