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On asymptotic limits of nonoscillations in functional equations with retarded arguments

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1. Introduction

Our main aim in this paper is to study the asymptotic properties of the nonoscillatory solutions of the differential equation

$$L_n y(t) + a(t)h(y(g(t))) = f(t), \tag{1}$$

where $n \geq 2$ and L_n is a disconjugate differential operator defined by

$$L_n y(t) = p_n(t)(p_{n-1}(t)(\cdots(p_1(t)(p_0(t)y(t))')'\cdots)')'. \tag{2}$$

The following conditions are always assumed to hold:

(i) $p_i \in C([\alpha, \infty), (0, \infty))$, $0 \leq i \leq n$, and

$$\int_\alpha^\infty p_i^{-1}(t)dt = \infty, \quad 1 \leq i \leq n-1; \tag{3}$$

(ii) $a, f, g \in C([\alpha, \infty), R)$, a is of one sign, there exists a $t_0 > \alpha$ such that $0 < g(t) \leq t$ for $t \geq t_0$, and $g(t) \to \infty$ as $t \to \infty$;

(iii) $h \in C(R, R)$, h is nondecreasing, and sign $h(y) = $ sign y.

We introduce the notation:

$$L_0 y(t) = p_0(t)y(t), \quad L_i y(t) = p_i(t)(L_{i-1}y(t))', \quad 1 \leq i \leq n. \tag{4}$$

The domain $\mathscr{D}(L_n)$ of L_n is defined to be the set of all functions $y: [T_y, \infty) \to R$ such that $L_i y(t)$, $0 \leq i \leq n$, exist and are continuous on $[T_y, \infty)$. In what follows by a "solution" of equation (1) we mean a function $y \in \mathscr{D}(L_n)$ which is nontrivial in any neighborhood of infinity and satisfies (1) for all sufficiently large t. A solution of (1) is called oscillatory if it has arbitrarily large zeros; otherwise the solution is called nonoscillatory.

It is well known [5, 7] that in case $L_n y(t) = y^{(n)}(t)$ equation (1) has a nonoscillatory solution with a prescribed limit as $t \to \infty$ if

$$\int^\infty t^{n-1}|a(t)|dt < \infty \tag{5}$$

and