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Localization of differential operators and holomorphic continuation of the solutions

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1. Introduction

The holomorphic continuation of solutions of linear partial differential equations across the multiple characteristic surfaces is the subject of this paper. In the preceding note [4] we attack this problem with the aid of the Goursat problem. Since the existence-domain of solutions of the Goursat problem is determined not only by the principal parts of the equations but also by their lower order terms, the results in [4] depend on the "*weighted principal parts*" of the operators which are not contained in the principal parts.

The purpose of this paper is to improve the results in [4] so that the theorems are also valid under the similar assumptions only on the principal parts of the operators and the properties of the boundary surfaces.

Let $P(z, \partial_z)$ be a linear partial differential operator with holomorphic coefficients defined near a point p in $\mathbf{C}^n$ and Ω be an open set with the C^2 boundary $\partial\Omega$ which contains p. Though the property of the holomorphic continuation is free from the choice of the local coordinates, we here employ the weighted local coordinates at p such that the normal direction z_1 of $\partial\Omega$ at p is assigned the weight 2, while the tangential directions $z_2,\dots, z_n$ are each assigned the weight 1. The motivation of this employment is that the boundary $\partial\Omega$ can be approximated by the quadratic hypersurfaces of the form

$$\operatorname{Re} z_1 = \operatorname{Re} \sum a_{ij}z_iz_j + \sum b_{ij}z_i\bar{z}_j.$$

To make this paper self-contained, some properties related to the weighted coordinates are restated in the next section which is the summary of the section 2 in [4]. In the third section the basic theorem is proved under some fixed local coordinates. The idea of the proof is due to Hörmander [1] and used also by Treves and Zachmanoglou to show the uniqueness of the Cauchy problem (see the references of [3]) and in [3] to obtain the holomorphic continuation theorem. The key point of this idea is to construct the family of surfaces which are non-characteristic with respect to $P(z, \partial_z)$ and cover a neighborhood of p. This basic theorem is the generalization of the theorem of the simple characteristic case. In the last section, § 4, we study the geometric conditions on $P(z, \partial_z)$ and $\partial\Omega$ to insure the existence of the local coordinates in the third section. The