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The L^p approach to the Navier-Stokes equations with the Neumann boundary condition

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1. Introduction

Let D be a bounded open set in R^n, $n\geq 3$, with smooth boundary S, and ν be the unit exterior normal to S. The motion of a viscous incompressible fluid in D is described by the Navier-Stokes equation:

$$(1)\qquad \begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + (u, \mathrm{grad})u + \mathrm{grad}\, q = f & \text{in } D\times(0, T), \\ \mathrm{div}\, u = 0 & \text{in } D\times(0, T), \\ u(x, 0) = a(x) & \text{in } D, \end{cases}$$

with the boundary condition:

$$(2)\qquad u(x, t) = 0 \quad \text{on} \quad S\times(0, T).$$

Here $u(x, t)=(u_1(x, t),\dots, u_n(x, t))$, $q(x, t)$ and $f(x, t)=(f_1(x, t),\dots, f_n(x, t))$ denote the velocity, the pressure and the external force respectively, and $(u, \mathrm{grad})= u_j\partial/\partial x_j$.

So far, the above problem has been attacked mainly within the framework of the Hilbert space $(L^2(D))^n$. In this framework the existence and uniqueness, local in time, of strong solutions were established, when $n=3$, by Kiselev and Ladyzhenskaya [9] under some regularity assumptions on the initial data. Then Kato and Fujita [5], [8] made these assumptions weaker and also proved similar but stronger results by the method of evolution equations in Hilbert spaces. Inoue and Wakimoto [7] extended the results of [5], [8] to the case when $n=4$, 5. But the case $n\geq 6$ still remains open.

On the other hand, in [5], Fujita and Kato suggested the possibility of removing the regularity assumptions noticed above by passing from L^2 to general L^p spaces. However, the existence of strong solutions in L^p spaces is still not known, mainly because of the lack of knowledge about the L^p-theory of the Stokes system, i.e. the linearized version of the problem (1) and (2).

In this paper we consider in $(L^p(D))^n$, $n<p<\infty$, the equation (1) under the following boundary condition (the Neumann condition for 1-forms, see [3]):