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Basic results on oscillation for differential equations with deviating arguments

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1. Preliminaries

This article deals with oscillatory and asymptotic properties of the solutions of differential equations with deviating arguments. The study is concentrated to differential equations of "fundamental" forms, since, in view of a comparison principle introduced by Staikos and Sficas [20, 21, 22], the results obtained can be extended for differential equations of more general forms in such a manner that is rather technical. More precisely, it is enough to treat here the n-th order differential equation with deviating arguments

$$(1) \qquad x^{(n)}(t) + a(t)\varphi(x[\sigma_1(t)],\dots, x[\sigma_m(t)]) = 0, \quad t \geqq t_0,$$

where the functions a, φ, σ_j $(j=1,\dots, m)$ are continuous and

$$\lim_{t\to\infty} \sigma_j(t) = \infty \quad (j = 1,\dots, m).$$

Most of the results here are concerned with the case where the function a is of constant sign. Then the equation (1) can be written in the form

$$(1)' \qquad x^{(n)}(t)I(a) + |a(t)|\varphi(x[\sigma_1(t)],\dots, x[\sigma_m(t)]) = 0, \quad t \geqq t_0,$$

where

$$I(a) = \begin{cases} +1, & \text{if } a \geqq 0, \\ -1, & \text{if } a \leqq 0, \end{cases}$$

is the so called *sign index* of the function a. For technical reasons, it is then more convenient to work with the differential inequalities

$$(1)'_{\leqq} \qquad x^{(n)}(t)I(a) + |a(t)|\varphi(x[\sigma_1(t)],\dots, x[\sigma_m(t)]) \leqq 0, \quad t \geqq t_0,$$

and

$$(1)'_{\geqq} \qquad x^{(n)}(t)I(a) + |a(t)|\varphi(x[\sigma_1(t)],\dots, x[\sigma_m(t)]) \geqq 0, \quad t \geqq t_0,$$

associated to the equation (1)′.

The function φ, or equivalently, the differential equation (1) and the differential inequalities $(1)'_{\leqq}$ and $(1)'_{\geqq}$ are said to be: